Valuing High Yield Bonds: a Business Modeling Approach

by

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Abstract

This paper proposes a valuation model of a bond with default risk. Extending from the Brennan and Schwartz real option model of a firm, the paper treats the firm as a contingent claim on the business risk. This paper introduces the “primitive firm”, which enables us to value firms with operating leverage relative to a firm without operating leverage. This paper emphasizes the business model of the firm, relating the business risk to the firm’s uncertain cash flow and its assets and liabilities. In so doing, the model can relate the financial statements to the risk and the value of the firm. The paper then uses Merton’s structural model approach to determine the bond value. This model considers the fixed operating costs as payments of a “perpetual debt”, and the financial debt obligations are junior to the operating costs. Using the structural model framework, we relative value the bond to the observed firm’s market capitalization, and provide a model that is empirically testable. We also show that this approach can better explain some of the high yield bond behavior. In sum, this model extends the valuation of high yield bonds to incorporate the business models of the firms and endogenizes the firm value stochastic process, which is a key element in high yield valuation in practice. We have shown that in relating the firm’s business model to the firm value, the resulting firm value stochastic process affects the bond value significantly.

Valuing High Yield Bonds: a Business Modeling Approach

A. Introduction

There has been much research in the valuation of corporate bonds with credit risks in the past few years. The impetus of the research may be driven by a number of factors. Recently there has been a surge of bonds facing significant credit risks, as a result of the downturn of the economy after the burst of the new economy bubble. For example, in the telecommunication sector, a number of firms have declared default because of the excess supply of telecommunication infrastructure and financial obligations. Another reason is the impending change in regulations in risk management. Increasingly regulators are demanding more disclosure of risks from the financial institutions and the measures of credit risks in the firm's investment portfolio. The financial disclosure would lead to the examination of the adequacy of capital for the firms. Finally, the credit risk model is important to the use of a number of recent financial innovations. These innovations include the collateralized debt obligations, credit default swaps and other credit derivatives that have demonstrated significant growths in the past few years. Credit risk model is important in determining these securities' values and managing their risks.

Valuing bonds with credit risk must necessarily be a complex task. A high yield bond tends to have the business risk of the bond's issuer. And, therefore, to value of a high yield bond may be as involved as valuing the equity of the issuer. Indeed, both the bonds and the equity of a firm are contingent claims on the firm value.

One approach to value a high yield bond is that of Merton (1974). The model views the firm's equity as a call option on the firm value and applies the Black-Scholes model to value a corporate bond. This approach does not require the investors to know the profitability of the firm and the market expected rate of return of the firm. The model only needs to know the prevailing firm value and its stochastic process. In essence, according to the Merton model, a defaultable bond is a default free debt embedded with a short position of a put option on the firm value, with the strike price equaling the face value of the debt and the time to expiration equaling the maturity of the bond. More generally, models that view a high yield bond as a bond with an embedded put option are called structural models.

There are many extensions of the Merton model. One general extension is the use of a trigger default barrier that specifies the condition for default. For example, the Longstaff and Schwartz[1] model (1995) allows the firm to default at any time whenever the firm value falls below a barrier. This approach views that a bond has a barrier option embedded in a default free bond. This model is extended by Saa-Requejo and Santa-Clara (1999) which allows for the stochastic strike price and Briys and de Varenne (1997) allow the barrier to be related to the market value of debt. Such extensions assume the stochastic firm value captures all the business risk of the firm. They do not model the business of the firm and they in particular ignore the importance of the negative cash flows of a firm in triggering the event of default.

[1] See p792, Assumption 4, Longstaff, F.A., and E.M. Schwartz, 1995, A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, *Journal of Finance,* Vol.50, No.3, 789-819.

To avoid such shortcomings, the Kim, Ramaswamy, and Sundaresan model (1993) assumes that the bondholders get a portion of the face value of the bond at default, which is based on the lack of cash-flow to meet obligations. They define the default trigger point as a net cash-flow at the boundary, when the firms cannot pay for the interests and dividends. Brennan and Schwartz use the real option approach to determine the firm value as a contingent claim on the business risk. Using this approach, they model the value of a mining company. The real option valuation approach extends the Merton model to specify the business model of a firm and therefore the approach values the corporate bonds as compound options on the business risks.

This paper takes this real option approach to value the high yield bonds. Specifically, we model the business of the firm and its operating cash flows contingent on the business risks. Using the structural model's compound option concept, we determine the default conditions of a firm, given its capital structure and the business model. In essence, our approach endogenizes the trigger default barrier of the firm using the firm's business model and the capital structure.

Specifically, we propose that firms' fixed operating costs play a significant role in triggering default of the bond's debt. When the firm has a negative operating income which cannot be financed internally, the firm must necessarily seek funding in the capital markets. However, if the firm value is low in relation to all the future financial obligations, then the firm may not be able to fund the negative operating income, leading to default. Indeed, some bonds are considered risky because of the firm's high operating leverage, even though the financial leverage may be low.

In comparing with the structural models in the research literature, our model suggests that the firm value stochastic process is not a simple lognormal process. Instead the firm value follows an "option" price process. And the debt is not a risk free bond embedded with a put option. It is embedded with a compound option and it is a "junior debt" to the fixed costs.

This approach has broad implications to debt valuation. Our model suggests that the pricing of defaultable bond must include more financial information of a firm, in particular, the financial and the operating leverage of the firm. The model allows for the firm to default before the bond maturity by allowing the negative cash flow to trigger a default. Since the model does not require an exogenously specified trigger default function, but solves for the default condition using the option pricing approach, we can use the model to price the bonds using the firm's financial statements, which are widely available. Therefore, the model can be tested empirically.

In this paper, we will provide some empirical evidence to support the validity of the model. While this paper provides a simple model, but we show that the approach is very general. Extensions of the model will be left for future research.

The paper proceeds as follows. We will describe the model in section B, presenting the assumptions made in the model. For the clarity of the exposition, in Section C, we provide a numerical example, showing how the model can be used to market available data. Section D presents some empirical evidence on the validity of the model. Section E discusses some of the implications of the model and, finally, Section F provides the conclusions.

B. Specification of the Model

This section presents the assumptions of the model. Similar to the Merton model, we assume that the market is perfect, with no transaction costs. We assume that there are corporate and personal taxes such that the assumptions are consistent with the Miller model. The corporate tax rate of the firm is assumed to be τ_c. In this world, the capital structure does not affect the value of the firm. We use a binomial lattice framework to construct the risk processes.

We assume that the yield curve is flat and is constant over time at an annual compounding rate of r_f. The bond valuation model is based on a real option model. Specifically, we begin with the description of the business risk of the firm by depicting the primitive firm lattice $V^p(n,i)$. We then build the firm value lattice $V(n,i)$, which includes some considerations of a valuation of a firm: fixed costs and taxes. Finally, we use the firm value lattice to analyze all the claims on the firm value, based on the Miller and Modigliani framework.

1. Primitive Firm

The firm, the equity, the debt, and all other claims on the firms are treated as the contingent claims to the primitive firm. Primitive firm is the underlying "security" to all these claims and it captures the business risk of the firm.

We begin with the modeling of the business risk. We assume that the firm has a fixed capital asset and the capital asset generates uncertain revenues. The gross return on investment (GRI) is defined as the revenue generated per $1 of the capital asset. GRI is a capital asset turnover ratio. In this simplified model, we consider a firm is endowed with a capital asset that does not depreciate and can generate perpetual revenues.

Specifically, we assume that GRI follows a binomial lattice process that is lognormal (or multiplicative) with no drift, a *martingale process,* where the expected GRI value at any node point equals the realized GRI at that node point (n, i), where n is the time steps and i denotes the state of the world. Specifically:

GRI(n+1 , i+1) = GRI (n, i)exp(σ

σ

$$GRI(n,i) = q \times GRI(n+1,i+1) + (1-q) \times GRI(n+1,i) \qquad (1)$$

where

$$q = \frac{1-e^{-\sigma}}{e^{\sigma}-e^{-\sigma}},\ \sigma \text{ is the volatility of the risk driver.}$$

We assume that the Miller and Modigliani theory can be extended to the multi-period dynamic model described above. In this extension, we assume that all the individuals make their investment decisions and trading at each node on the lattice. These activities include the arbitrage trades described in the Miller and Modigliani theory. The results of the theory apply to each node. Therefore, there is a cost of capital ρ

ρ

present value of all the firms' free cash flow along all the paths on the lattice. In particular, the lattice of primitive firm value is given as,

$$V^p(n,i) = \frac{CA \times GRI(n,i) \times m}{\rho} \tag{2}$$

where *m* is the gross profit margin.

By the definition of the binomial process of the gross return on investment, we have

$$V^p(n+1, i+1) = V^p(n,i)e^{\sigma}. \tag{3}$$

Further, since the cost of capital of the firm is ρ

$C_u = V^p \;\; n \;\; i \;\; \times \rho \times e^{\sigma}$ Therefore the total value of the firm V_u^p, an instant before the dividend payment in the upstate is

$$V_u^p = V^p \times (1+\rho) \times e^{\sigma}. \tag{4}$$

Similarly, the total value of the firm V_d^p, an instant before the dividend payment in the downstate is

$$V_d^p = V^p \times (1+\rho) \times e^{-\sigma}. \tag{5}$$

Then the risk neutral probability p is defined as the probability that ensures the expected total return is the risk-free return.

$$p \times V_u^p + (1-p) \times V_d^p = (1+r_f) \times V^p . \tag{6}$$

Substituting V^p, V_u^p, V_d^p into equation above and solve for p, we have:

$$p = \frac{A - e^{-\sigma}}{e^{\sigma} - e^{-\sigma}} \tag{7}$$

where

$$A = \frac{1+r_f}{1+\rho} .$$

Note that as long as the volatility and the cost of capital are independent of the time n and state i, the risk neutral probability is also independent of the state and time, and is the same at each node point on the binomial lattice. We have now changed the measure from market probability to the risk neutral probability. We will use this risk neutral probability to determine the values of the contingent claims.

3. *The Firm Value*

We assume that the firm pays out all the free cash flows. Let the fixed cost be FC, which is constant over time and state. In the case of negative cash flow, we assume that the firm gets tax credits, and the firm raises the funds from equity. This assumption is quite reasonable since tax credits can carry forward for over 20 years, the government in essence participates in the business risks of the firm and it should not affect the basic insight of the model. The cash flow is the revenue net of the operating costs, fixed costs and taxes;

$$CF(n,i) = \left(CA \times GRI(n,i) \times m - FC\right) \times (1-\tau)\,. \tag{8}$$

This model assumes that the firm has no growth over this time horizon. This assumption is quite reasonable because the high yield companies often cannot implement growth strategies. Further, the model can be extended in a straightforward manner to incorporate growth for firms that growth is important to its bond pricing. Ho and Lee (2004) provides an extension of the model with growth, allowing for optimal investment decisions.

The terminal value at each state in the binomial lattice at the horizon date has four components: the present value of the gross profit, the present value of the fixed costs that takes the possibility of future default into account, and the present value of the tax which is approximated as a portion of the pretax firm value, and finally, the cash flows of the firm at each node point.

Following the Merton model (1974), we assume that the firm pays no dividends after the planning period and the primitive firm follows a price dynamic described below.

$$dV^p = \rho V^p dt + \sigma_p V^p dZ \tag{9}$$

where dZ is the wiener process.

The present value of the fixed costs is determined as a hyper-geometric function, according to Merton (1974), since we assume that the firm can go default in the future and the fixed costs are not paid in full.

The lattice of the firm value is determined by rolling back the firm values, taking the cash flows into account. The firm value at the terminal period at each node is

$$V(n,i) = Max\left[\left\{\frac{CA \cdot GRI(n,i) \cdot m}{\rho - g} - \Phi\left(\frac{CA \cdot GRI(n,i) \cdot m}{\rho - g}\right) + \left(CA \times GRI(n,i) \times m - FC\right)\right\}(1-\tau_c),\ 0\right] \quad (10)$$

where Φ(g) is the present value of the perpetual risky fixed cost, and Φ(g) is the valuation formula of the perpetual debt given by Merton(1973) presented in the Appendix.

In the intermediate periods, the firm value is determined by backward substitution,

$$V(n,i) = Max\left[\frac{p \times V(n+1,i+1) - (1-p) \times V(n+1,i)}{(1+r_f)} + \left(CA \times GRI(n,i) \times m - FC\right) \times (1-\tau_c),\ 0\right]. \quad (11)$$

4. Debt valuation and the Market Capitalization

We assume that the firm value is independent of the debt level. The value of the bond is determined by the backward substitution approach. The stock lattice is the firm lattice net of the bond lattice. We first consider the terminal conditions for the bond to be

$$Min[debt\ obligation\ at\ T,\ firm\ value\ at\ T].$$

We then conduct the backward substitutions, such that we apply the valuation rule at each node point:

$$Min\ [backward\ substitution\ bond\ value + bond\ cash\ flow,\ firm\ value].$$

Following the standard methodology, the rolling back procedure leads to the value of the debt at the initial value. The market capitalization of the firm is the firm value net of the debt value.

C. A Numerical Illustration

We assume that the yield curve is flat and is constant over time at 4.5% annual compounding rate. The market premium is defined as the market expected return net of the risk-free rate, which is assumed to be 5%. The tax rate of the firm is 30%, which is assumed to be the marginal tax rate. The model is based on a 5 steps binomial lattice. It is a one factor model, with only the business risk. The model is arbitrage-free relative to the underlying values of a firm that bears all the business risks of the revenues.

We use one firm, Hilton Hotels, in the sector of consumer and lodging, as an example to illustrate the implementation of the model. On the evaluation date October 31, 2002, the market capitalization is reported to be $4,944 million, and the stock volatility estimated to be the one-year historical volatility of Hilton's stock is 51.9% and the stock beta is 1.255. Using the capital asset pricing model, we estimate that the expected rate of return of the stock $r = 4.5\% + 1.255\times5\%=10.775\%$.

The financial data is given from the financial statements as follows. The revenue is $2,834 million, with the operating cost of $1,542 million, and fixed cost of $726 million. The capital asset is $7,714 million with the long term debt $5,823 million and interest costs $357 million. Using this data, we can calculate

Gross Return on Investment (GRI) = Revenue/Capital asset= 0.367

Profit margin (m) = (Revenue-operating cost)/Revenue= 45.58%

1. The GRI Lattice

To generate the GRI lattice, we have to assume the sector cost of capital and the volatility initially to begin the iterative process. Let us assume the cost of capital spread be 1.57% and the sector volatility 11.54%. The assumed data is an initial input for the non-linear optimization procedure. Using the assumed data, we can calculate the expected returns of the sector (ρ

ρ

$$V^p \; n \; i \; = \frac{CA \times GRI \;\; n \; i \;\; \times m}{\rho}$$

We can also calculate the cash flow of the primitive firm at each node point. The lattice of cash flow at each state, based on the capital asset of $7,714 million;

$$CF^p(n,i) = CA \times GRI(n,i) \times m \,. \qquad (12)$$

Risk neutral probability: $p(n,i) = \dfrac{\dfrac{(1+r_f)}{(1+\rho)} - e^{-\sigma}}{e^{\sigma} - e^{-\sigma}}$ = 0.414223

2. The Firm Value

The fixed costs of the firm stay the same. Gross margin also remains constant. The firm defaults when the firm cannot finance the fixed costs. All excess cash flows are paid out.

Lattice of cash flow at each state: at each node, the cash flow is the revenue net of the operating costs, fixed costs and taxes; $CF(n,i) = (CA \times GRI(n,i) \times m - FC) \times (1-\tau)$.

The terminal value at each state is given below. By assuming that the long term growth rate g be 1.57%, the firm value at the terminal period at each node is

$$V(n,i) = Max\left[\left\{\frac{CA \cdot GRI(n,i) \cdot m}{\rho - g} - \Phi\left(\frac{CA \cdot GRI(n,i) \cdot m}{\rho - g}\right) + \left(CA \times GRI(n,i) \times m - FC\right)\right\}(1-\tau),\ 0\right]$$

where $\Phi(g)$ is given in the Appendix. At the terminal date, the time horizon where n=5, the firm value for each node is:

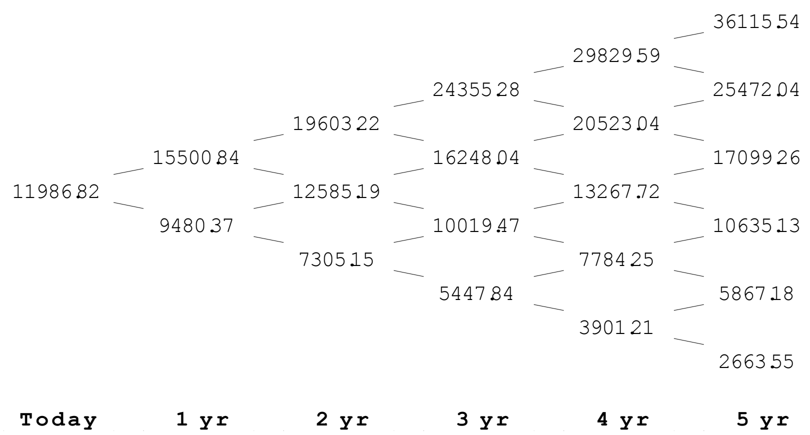
State (i)	0	1	2	3	4	5
Firm value($mil)	2663.545	5867.177	10635.13	17099.2615419.2	25472.04	36115.54

Now we roll back the firm value from the terminal value, taking the cash flows into account. In the intermediate periods, the firm value is determined by backward substitution,

$$V(n,i) = Max\left[\frac{p \times V(n+1,i+1) - (1-p) \times V(n+1,i)}{(1+r_f)} + \left(CA \times GRI(n,i) \times m - FC\right) \times (1-\tau),\ 0\right].$$

The resulting lattice is given by:

Figure 6 Lattice of the firm value



3. Debt valuation and the Market Capitalization

The debt structure of Hilton Hotels is somewhat complicated with eight bonds, given below:

Table 1 Debt package of Hilton Hotels

Obseved date	Maturity	Coupon rate	Principal	Price	# of outstanding
20030228	20130228	0.061	1000	100	3473000
20030228	20060515	0.05	1000	96	500000
20030228	20091215	0.072	1000	99	200000
20030228	20171215	0.075	1000	92	200000
20030228	20080515	0.076	1000	102	400000
20030228	20121201	0.076	1000	99	375000
20030228	20070415	0.08	1000	104	375000
20030228	20110215	0.083	1000	105	300000

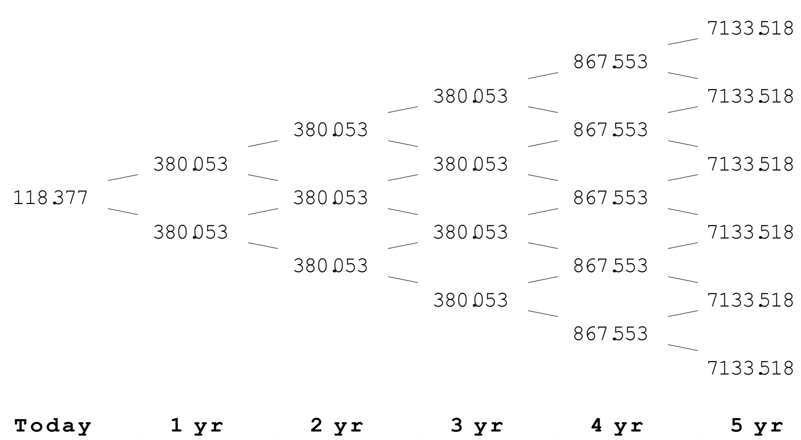
Using the debt structure given above, we can calculate the promised cash flow of the bonds, which are $118.533 thousands at year 0, 380.053 thousands for the period from year 1 to year 3, $867,553 and $7133.518 thousands at year 4 and 5 respectively.

Time	0	1	2	3	4	5

Debt cash flow	118.533	380.053	380.053	380.053	867.553	7133.518

Now, we use the lattice of the firm model and the lattice of the debt cash flow, and use the standard backward substitution procedure to determine the debt value. The resulting lattice of the debt is given below.

Figure 8 Lattice of debt value



Now, the market capitalization value is the firm value net of the debt value.

Finally, given the bond price to be $5,818 million, the internal rate of return is 9.26% or the credit risk spread, which is the internal rate of return net of the risk free rate, is 476 basis points. From the two nodes of the first period of the stock lattice and the sector lattice, the lattice of the primitive firm, we can calculate the stock volatility and the sector volatility.

4. *Calibration*

Recap that this procedure thus far has assumed the following input data: long-term growth rate of the firm g, sector volatility σ_p, sector expected excess return ρ

×stock volatility/sector volatility (13)

Therefore, we can use a non-linear estimation procedure in perturbing the assumed data, the long term growth rate and the sector volatility, such that the model derived value equals to the observed values.

Market capitalization and stock volatility = observed market capitalization and stock volatility.

Market Capitalization	4944.6	Given Stock vol	0.51898
Calibrated Stock Value	4944.6	Estimated Stock vol	0.51925

D. Empirical evidence

We use an example of McLeodUSA to demonstrate empirically the relationship between the market capitalization and the bond package value. We show that as the market capitalization of the firm falls, the bond package value would also fall. But, the bond value falls more precipitously than the market capitalization as the firm approaches bankruptcy.

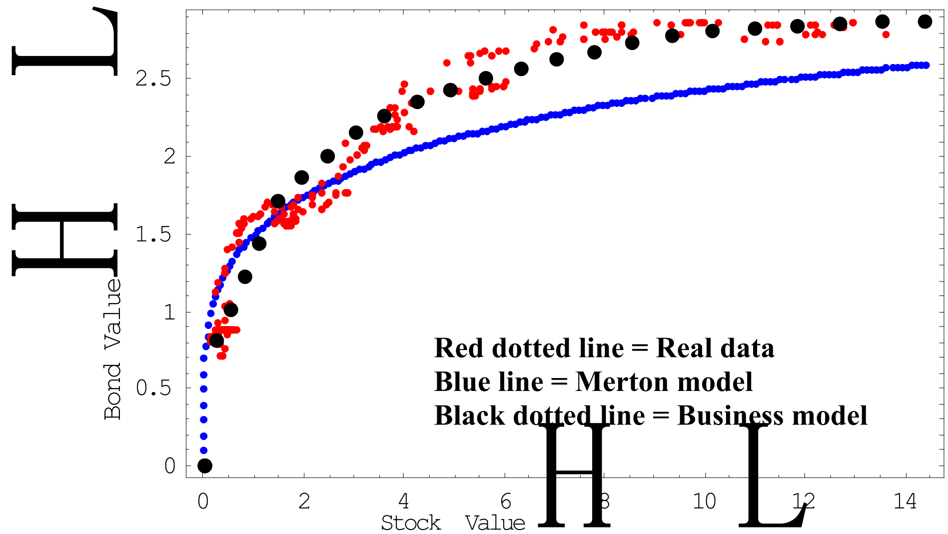
We also show that the Merton model of the corporate bond can explain much of this relationship between the market capitalization and the bond value. But the model tends to understate the acceleration of the fall in the bond prices when the market capitalization falls below certain value.

We use our model to explain this relationship between the market capitalization and the bond package value. Specifically, we assume that the gross return on investment declines, leading the fall in the market capitalization and the bond value. It is quite reasonable to make this assumption. A source of the financial problem of McLeodUSA was the expectation of the demand for communication had been falling along with an excess supply of the communication networks. As a result, the market revised their expectations of the returns of the assets of McLeodUSA, and hence the fall of GRI.

We use the calibrated business model of McLeodUSA. The sector volatility and the implied long term growth rate are estimated to be 0.0935 and 0.0332 respective. Further we add a spread to Treasury curve in discounting the bond such that the bond price equals the observed price initially. The use of a spread is reasonable because we have discussed that bonds tend to have a liquidity spread and a risk premium (a market price of risk for the model risks not captured by the model) in predicting defaults. Referring to Figure 1, the results show that the proposed model provides relatively better explanatory power of the observations than that of the Merton model. In particular, using the proposed model, we predict the precipitous fall of the bond value better than that of the Merton model.

The empirical evidence can be explained intuitively. McLeodUSA is a communication company that has significant operating costs. Note that, the debts outstanding do not mature in the next three years and there is no immediate maturity of crisis for the firm in the short run. However, because of the significant fixed cost resulting in losses of the operation, the firm's operation has to be supported by external financing. When the market revised downwards on the returns of the assets, the market capitalization falls leading to the situation where McLeodUSA fails to have access to the capital market to fund its negative operating costs. In essence, the firm defaults on its operating costs, something that the Merton model would not have considered.

Figure 10 Stock value vs. Bond value of McLeodUSA



E. Implications of the Model

The proposed model is an extension of the Merton's structural model, where we have use the relative valuation approach to solve for valuation of the market value of debt. And therefore, we have not used any historical estimation of the survival rate of the bonds, which require the model to hypothesize the appropriate market discount rate for the expected cash flow. Also this approach does not require any assumption of the recovery ratio, which is endogenous in the structural model.

When compared with the Merton model, this model predicts that the firm can go default without the crisis of maturity. The default event can be triggered by the fixed costs. This model can explain the market observation that some firm has low credit rating and traded with a high yield spread, even though the market debt to equity ratio is low.

This model of the bond price is sensitive to the stock price like the Merton model. The main advantage of the proposed model is to allow us to use more detail information about the firm, and therefore the bond valuation model is more realistic. Finally, the model is empirically testable. The financial data, bond yield spreads and stock prices are relatively accessible.

F. Conclusions

This paper proposes a valuation of bonds with credit risks. The main contribution of the paper is to introduce the concept of a primitive firm. The primitive firm has neither operating leverage nor financial leverage. Given the risk class of this primitive firm, we can determine the value of a firm with operating and financial leverage, as a contingent claim to the primitive firm.

This approach enables us to introduce the business model of a firm in the high yield bond valuation. Relating the high yield bond pricing to the firm's business is standard in practice. And this approach provides a more robust analytical framework to the practitioners in the high yield area.

The model provides intuitive insights into the high yield bond behavior. For example, the model shows that the fixed costs of the firm can be viewed as the "perpetual debt" of the firm, senior to the financial debt obligations. Such a treatment of the fixed costs significantly affects the probability distribution of the event of default of a firm and the valuation of the debt. Furthermore, it shows that the bond price is more sensitive to the market capitalization. As the market capitalization falls, the firm with significant fixed costs would fall precipitously with the market capitalization, before the event of default becomes imminent. For this reason, the model shows that the relationship of the probability of default and the bond price must also dependent on the fixed cost level. This may explain the observation that the rating of the bond tends to lag the market pricing of the bonds. Since the bond rating measures the probability of default, which is different to the bond pricing, as this model shows, the extent of the lag must depend on the firm's operating leverage.

This also suggests that models that use bond value or market capitalization to predict the probability of default can also be erroneous. We show that the bond value depends on both the recovery ratio and the probability of default. Since the recovery ratio depends on the firm's operating leverage, the firm default probability cannot be related simply to the market capitalization or the bond price, without taking the firm's business model into account.

Finally, the proposed model shows that the use of a primitive firm can has broad implications to future research. The study of the primitive firms for different market sector will enable us to better understand the high yield bond valuation.

Appendix

The valuation formula of the perpetual debt is given by Merton (1973).

$$\Phi(V,\infty)=\frac{FC}{r_f}\left\{1-\frac{\left(\frac{2FC}{\sigma^2 V}\right)^{\frac{2r_f}{\sigma^2}}}{\Gamma\left(2+\frac{2r_f}{\sigma^2}\right)}M\left(\frac{2r_f}{\sigma^2},2+\frac{2r_f}{\sigma^2},\frac{-2FC}{\sigma^2 V}\right)\right\} \tag{A.1}$$

where

$V =$ the primitive firm value
$FC =$ fixed cost per year
$r_f =$ risk free rate
$\Gamma =$ the Gamma function (defined in the footnote)
$\sigma =$ the standard deviation of $\tilde{G}RI$
$M(\bullet) =$ the confluent hypergeometric function (defined in the footnote)

$$M\left(a,2+a,-\frac{2FC}{\sigma^2 V}\right)=\frac{1}{br_f}e^{-\frac{b}{V}}\left[-(1+a)bFC\left(\frac{b}{V}\right)^a+e^{\frac{b}{V}}FC\left(aV\Gamma(2+a)+(1+a)(b-aV)\Gamma\left(1+a,\frac{b}{V}\right)\right)\right]$$

where

$$a=\frac{2r_f}{\sigma^2},\ \ b=\frac{2FC}{\sigma^2},\ \ \Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt,\ \text{and}\ \Gamma(a,x)=\int_x^\infty t^{a-1}e^{-t}dt$$

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