# Valuing Fixed Rate Mortgage Loans with Default and Prepayment Options

by

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**Abstract**

There are three integral components to value a fixed rate mortgage loan: (1) the mortgagors' American straddle option on the underling loan, (a call option to refinance and a put option to default); (2) the mortgagors' heterogeneous behavior in exercising the options inefficiently; and (3) the market price of risk, (the option adjusted spread (OAS)). Despite the dominance of mortgages in the capital market, scant research considers the valuation of mortgage loans while taking these three components into consideration, a contribution of this paper. Specifically, this paper uses a multinomial logit model to describe the mortgagors' behavior in dealing with the competing refinancing and default risks, and then utilizes a two factor arbitrage-free interest rate model to value the mortgages.

The paper shows that the prepayment-default model has significant explanatory power. Using the mortgage loan prices at origination, the model shows that OAS and duration depend on the FICO score, original loan-to-value ratio, the loan size and the recovery ratio. Lastly, a model of the economic value of a loan default guarantee is specified and the model shows that the price elasticities of the guarantee with respect to the loan size and the borrower's FICO score are -0.46 and -11.89 respectively.

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### 1. Introduction

The mortgage market in the United States is broadly based and vertically deep. From the most general perspective, we can separate the primary mortgage market from the secondary market. The primary mortgage market originates loans to borrowers (the "mortgagors") and sells the loans into the secondary market for securitization. Not all mortgage loans are sold into the secondary market for securitization. In this case, banks or financial institutions in general, are direct investors in these mortgage loans.

The traditional 30 year fixed rate mortgages contain a call option to prepay without penalty and the put option to default. Mortgage lenders are selling embedded American straddles (combined call and put options) to the mortgagors. However, the mortgagors do not exercise these options efficiently, and, moreover, their behavior is heterogeneous and cannot be represented by a typical mortgagor. The average refinancing and defaulting behavior of a portfolio of mortgages has to be described in terms of the age of the mortgage, seasonality, burnout level, underwriting standards, and credit scores in order to value the mortgages. Such risks have to be priced into the valuation of the mortgage. Therefore, the option adjusted spread has to be estimated to determine the market pricing of the risks embedded in these mortgage loans. In sum, valuation of mortgage loans must take into consideration three salient features: (1) the embedded options, (2) inefficiency in exercising the options, and (3) the market price of mortgage loan risks.

Despite the importance of the subject, scant research papers address all three salient features. Chen (1996), Levin(2005), Cheyette (1996), and Longstaff (2002) determine the call option by the optimal exercise rule, incorporating a measure of refinancing inefficiency. However, these papers fail to model aging, seasonality and other salient features of the mortgagors' heterogeneous behavior into account. While Schwartz and Torous (1989, 1992, 1993), Boudoukh et al (1997), and Hayre (2001) use econometrically estimated prepayment models to capture the factors affecting the mortgagors' behavior in prepayment, they ignore defaults. They fail to identify default and prepayment as separate events, thus missing the insights into the mortgagors' behavior that provide a more accurate valuation model, even for mortgage securities with full credit guarantees.

Incorporating default risk in mortgages is challenging. There is voluminous literature on valuing defaultable bonds, yet sparse literature on mortgages. Credit risk literature on bonds uses the structural model and the reduced form models to value the default risk. However, such approaches cannot be adapted to value mortgage loans because of the complex interactions of the prepayment and default risks with the mortgagors' heterogeneous behavior. In addition, underwriting requirements and the borrower's behavior in the case of mortgage loans are strictly different than corporate bonds.

A second genre of mortgage termination research is inferential (and normative) in nature. It attempts to take the perspective of the borrower toward asking the question, 'why do borrowers default or prepay their mortgage? For example, there is a growing literature in modeling the competing prepayment and default risks. Downing et al. (2002) focuses on the housing index to specify the structural model in default risk. Calhoun and Deng (2002), Deng, Quigley and Van Order (2000), and Dunsky and Pennington-Cross (2003) present a method to estimate the mortgagors' behavior with these competing options. However, these papers fail to consider the question of mortgage valuation.

One contribution of this paper is that we model the termination risks from the borrowers' perspective and use the model to value the mortgages. We propose a valuation model of 30 year fixed rate mortgage loans, which uses a multinomial logit model to specify the mortgagors' behavior and a two factor arbitrage-free interest rate model to specify the interest rate movements. The model also specifies the determinants of the option adjusted spread for mortgage loans using the mortgage origination information.

Another contribution of the paper is to provide insights into the question: given the conceptual structure of the econometric model, what factors account for the default event and the prepayment event? At the borrower level of investigation, we cannot overlook the competing risks of the nature of the default and prepayment events. In particular, during each month of the mortgage, the borrower must choose between making their scheduled mortgage payment, prepaying their mortgage (owing to refinance, or move) or failing to make the scheduled payment. The default and prepayment events are mutually exclusive in that the borrower is unable to default on a prepaid loan or prepay on a defaulted loan.

The main empirical results based on our sample data and valuation model are as follows. (1) While the mortgage age, seasonality, burn-out factor, and credit score at origination are important factors affecting both the prepayment and default experiences, their impacts are different. The results highlight the competing nature of the two options, and the importance of separating the conditional default rate from the conditional prepayment rate. The model can provide reasonable explanatory power for both the conditional prepayment rate and the conditional default rate. (2) The prepayment model is shown to be reasonable in fitting the historical prepayment and default experiences.

The valuation model is then analyzed with the following three main results. (1) The option adjusted spreads vary significantly across the loans, suggesting that the market price of mortgage loan risk is not necessarily cohort or pool specific, but rather borrower/loan specific. Our results show that if the recovery ratio is between 90%-95%, the OAS at origination would not tighten or widen with the FICO scores above 700. However, for lower FICO scores, the option adjusted spread is sensitive to the assumed recovery ratio in the valuation. This result is important to determine the value of a mortgage loan where there is no active securities market to provide the market price. The OAS model can be used instead. (2) The economic cost of guaranteeing the default risk depends primarily on the principal amount borrowed, the loan to value ratio and the FICO score. The guarantee price elasticities to the original loan size and the FICO scores are -0.44 and -11.89 respectively. The guarantee model is useful for any financial institutions which provides credit risk guarantees and for banks to hedge their credit positions. And (3) the duration of a mortgage loan is then studied over a range of recovery rates. Our findings indicate that high OLTV loans originated to low credit score borrowers will have shorter duration relative to all other mortgages when fully guaranteed. However, for an investor in whole loans with an expected recovery rate of 80%, this result does not hold. This result has broad implications to banks and asset management in managing the interest rate risks of a risky mortgage portfolio.

The paper proceeds in five sections. The next section describes the empirical implementation of the model. Section 3 describes the empirical results of the prepayment/default model. Section 4 provides the results of valuing the mortgage loans. And section 5 contains the implications and conclusions.

## 2. Empirical Implementation

We introduce some new independent variables to the standard prepayment-default modeling literature. The following subsection describes these novel independent variables. The discussion is separated into two groups; factors included into both equations and factors only contained in the prepayment equation. Following the discussion of the prepayment-default factors the estimation methodology and the valuation methodology are discussed. The final subsection reviews the study's data.

### *2.1.* ***Joint Default and Prepayment Factors***

There are four sets of variables that are common to both the default and prepayment equations. The first two variable sets are designed to capture seasoning and seasonality of the borrower's repayment behavior. Seasoning or the baseline function captures borrower's behavior through time as measured in months. The baseline function is specified as a spline function with twelve knot or inflection points[1]. Seasonality captures the variation in borrower's behavior throughout the calendar months (January to December). Seasonality is captured with a series of 11 dummy variables with the excluded month as January. As expected, the panel of loan histories is quite uniformly distributed through the calendar.

The next set of variables included in both equations capture the borrower's credit worthiness and loan characteristics at origination. Jointly, credit scores, the original loan

[1] The baseline function or loan seasoning is specified as a spline of the number of months that the loan is making scheduled payments. Relative to the alternative baseline functions (e.g. flexibly parametric or functions of age and age squared) we believe that the spline specification is a nice balance between parameter parsimony yet still offers flexibly to the model. Twelve spline knots are chosen to capture this seasoning behavior. The estimated parameters capture the slope between the respective knot points. The knot points are chosen non parametrically such that more spline knots in regions of greater support.

to value (OLTV) ratio and the size of the loan capture a borrower's ability to repay the mortgage. For a given OLTV, large mortgages require greater down payments. OLTV, loan size and credit scores are incorporated into the model in the form of spline functions. The FICO scores are scaled by 100, such that the smallest value is 4.00 and the largest value is 8.50. Unlike the baseline or seasoning function, the spline knots for the loan size and OLTV are chosen at expected kinks in the household demand function for mortgage debt (Dunsky and Follain, 2000) and standard underwriting requirements.

Two additional sets of variables are only included in the prepayment equation: refinance function and the yield curve spread. The yield curve spread measures the slope of the yield curve from the two year maturity to the ten year maturity. The refinance function is constructed to capture the borrower sensitivity to changes in market rates toward prepayment. The refinance function is specified as the interaction between a refinance spread and a burnout measure, a feature not modeled in standard literature. The burnout factor captures the difference in the refinancing efficiency between two otherwise identical loans that have gone through different historical interest rate experiences. Note that burnout is most commonly measured in terms of a pool or cohort of mortgages - our broader interpretation here focuses on the number of missed refinance opportunities over a fixed transaction cost threshold. The refinance function is defined as $refinance\_function_{i,t} = refinance\_spread_{i,t} * burn\_out_{i,t}$, where

$$refinance\_spread_{i,t} = \left( \frac{PMMS_{i,t=0}}{PMMS_{i,t}} \right).$$

The refinance spread, is the ratio between the Primary Mortgage Market Survey (PMMS) rate for the $i^{th}$ loan at origination (*t=0*) to the current period PMMS rate. The PMMS rate is the current mortgage rate at time t.

There are two motivations to specify the ratio in terms of the PMMS. The primary goal of the refinance ratio is to measure the opportunity cost between refinancing at the current market rate and paying the fixed contract rate on the existing loan. It is a common practice to define this ratio in terms of the contract rate relative to the market rate. However, since the borrower can select from a menu of rates and points conditional on their credit score, the former measure contains noise for our purpose. More specifically, original coupons are positively related to the credit risk level and inversely related to the number of points paid at origination. Both the credit spread and points paid are interpreted as noise since our primary goal is to capture the borrower's response to changes in the market interest rate for their prepayment decision.

The burn out factor is defined in terms of the significantly positive refinance spread cumulated over the age of the mortgage, reflecting missed refinance opportunities. As such,

$$burn\_out_{i,t} = \sum_{t=0}^{T} MAX\left( \frac{PMMS_{i,t=0}}{PMMS_{i,t}} - 1.1, 0 \right) \qquad (2)$$

where 1.10 is assumed as the refinance transaction cost. That is, we assume that a refinance opportunity occurs whenever the original PMMS rate exceeds the prevailing PMMS by 10%.

Given a set of knot points, the *refinance_spread* is specified as a linear spline function. The function should have the qualitative behavior of an S curve: when the refinance – ratio is low, there is a constant base refinancing rate. As the refinance_spread increases, the refinancing rate also increases. But when the refinance-spread exceeds a certain level, the refinancing rate should remain stable, at a high level. However, the behavior of this *S* function varies with the burnout level. Our refinance function estimates *n S* functions as we categorize the sample into *n* buckets by the burn out function. The function is presented in Appendix A.

### *2.2. Default-Prepayment Model and the Valuation Methodology*

The Default-Prepayment model is a Multinomial Logit (MNL) model. MNL models are commonly deployed to model competing risk events in mortgage termination literature. One property of the MNL is the independence between observations. For example, given a panel or event history of a loan from origination to termination, each monthly observation is treated as though it were independent from the prior observation. A second property of the MNL is that the sum of the three events (prepay, default, no exit) must sum to one. Increases in one risk must be offset by decreases elsewhere, if the risks are truly competing.

MNL treats the dependent variables as a polytomous qualitative choice variable. Clapp et al (2000 and 2001) shows that the bivariate logit (MNL with two outcomes) provides a convenient method for dealing with prepayment risk and default risk. The information for each loan is restructured to include one observation for each time period in which that loan is active.

Specifically, the monthly conditional prepayment rate (CPR) and the monthly conditional default rate (CDR) are defined as $pp_{i,t}$ and $dft_{i,t}$ respectively for the $i^{th}$ loan in the $t^{th}$ month.

$$pp_{i,t} = \left( \frac{\exp(x_{i,t}'\beta_{pp})}{1+\exp(x_{i,t}'\beta_{pp})+\exp(x_{i,t}'\beta_{dft})} \right) \quad (3a)$$

and

$$dft_{i,t} = \left( \frac{\exp(x_{i,t}'\beta_{dft})}{1+\exp(x_{i,t}'\beta_{pp})+\exp(x_{i,t}'\beta_{dft})} \right) \quad (3b)$$

and $x_{i,t}$ are the variables and the vectors of estimated parameters ( $\beta_{pp}$ and $\beta_{dft}$ ) are described in the empirical section.

The log likelihood function is given by

$$\ln L(\beta_{pp}, \beta_{dft}) = \sum_i \sum_t \alpha \ln pp_{i,t} + (1-\alpha) \ln dft_{i,t} \quad (4)$$

where $\alpha = [0,1]$, 1 is an indicator variable depending whether the event is default or prepay.

The MNL is estimated using maximum likelihood by treating restructured discrete time information for each loan taken from identical and independent distributions. This estimation assumes irrelevant alternatives, where the elimination of one of the choices should not change the ratios of probabilities for the remaining choices. The estimation further assumes that the borrower's prior choices at any point in time are independent of those at any other point in time.

Equation (4), $\ln L(\beta_{pp}, \beta_{dft})$ is estimated from the loan level panel data, which we will discuss in the following section. Given a set of scenarios of yield curve evolutions generated from an arbitrage-free interest rate model, the prepayment-default model provides a set of cash flows of a mortgage loan corresponding to the interest rate scenarios. These cash flows are used to determine the mortgage loan value. We now proceed to describe the valuation procedure in more detail.

In this paper, we use the two factor arbitrage-free Generalized Ho-Lee Model (2005). The model is provided in Appendix B for completeness of the paper. This model is chosen for a number of reasons. The model is arbitrage-free, and therefore, the valuation is consistent with the market observed yield curve and the swaption prices that determine the implied volatilities of the interest rates. The interest rate model is calibrated to the market treasury curve and the swaption volatility surface. Calibration to the swaption prices using over 90 prices for each sample date. Being a two factor model, the model can generate a broad range of yield curve shape, unlike the one factor model. The model uses the implied volatility functions that are based on implied correlations of rates, resulting in a more parsimonious specification of the model (see Ho and Mudavanhu (2006)) providing accurate pricing at the same time. The mix normal and lognormal interest rate model truncates extremely high and low interest rates, and describe the mean reversion behavior of the interest rate movements.

Given the two factor interest rate model, we now generate random interest rate paths in the two dimension lattice, with monthly step size. At each node point of the lattice, we calculate the one month rate, two year, and 10 year rates from the arbitrage-free interest rate model.

We use the 2-year and 10-year rates represented by $r_2$ (t) and $r_{10}$ (t) respectively to determine the PMMS(t) rate based on the empirical model, estimated using the historical experience.

$$PMMS(t) = 2.638 + 0.0706\ r_2(t) + 0.7576\ r_{10}(t) + \varepsilon(t) \quad (6)$$

PMMS(t) is used in the *refinance_spread* and *burn_out* factors defined in equations (1) and (2). The generated ten and two year note rates are also used to construct the yield curve spread variable. The slope of the yield curve is defined as the 10 year rate net of the 2 year rate. Given these inputs to the prepayment-default model, we can now determine CPR(t) and CDR(t) at each node point. The prepayment-default model provides the probability of each event. The expected cash flow for a particular interest scenario is calculated as follows.

For a particular $i^{th}$ loan with maturity T (in months) and monthly coupon rate *c*, let P(t) be the remaining principal amount and Y(t) the scheduled payment of the loan at time t = 0, 1, … T, where T is the stated maturity date for $1 principal. For clarity, we suppress the index i for each loan. In specifying the model, we are referring the model for a representative loan. Since the mortgage loans are fixed rate amortization bond, the scheduled payments are determined as below.

$$Y(t) = 1/\left( 1/(1+c) + 1/(1+c)^2 + \ldots + 1/(1+c)^{T-t}\right). \qquad (7)$$

The scheduled principal paydown at time t is given by:

$$\text{Scheduled principal paydown} = (Y(t) - c)\, P(t) \qquad (8\ a)$$

$$\text{The unscheduled principal paydown} = (CPR(t) + CDR(t))\, P(t) \qquad (8\ b)$$

The total change in principal over one period is:

$$P(t) - P(t+1) = (Y(t) - c)\, P(t) + (CPR(t) + CDR(t))\, P(t) \qquad (9)$$

In simplifying, we can determine the remaining principal recursively by:

$$P(t+1) = (1 + c - Y(t) - CPR(t) - CDR(t))\, P(t) \qquad (10)$$

The pay down of the principal of a mortgage portfolio is not the same as the principal payments to the lenders, the banks. This is because at default, the lenders receive only a portion of the promised principal. The proportion of the received amount to the promised principal is referred to the recovery ratio, [illegible] where 0 < [illegible] < 1. Our model does not seek to model the recovery ratio; therefore we simulate the valuation using a range of recovery ratios. For the guaranteed pool value, the recovery ratio is 1, as the investors cannot distinguish between prepayments and defaults.

Given these assumptions, the cash flow to the bank conditional on neither unscheduled prepayment nor default would be P(t) Y(t). Conditional on default, the payment is CDR(t) [illegible] P(t), and conditional on an unscheduled prepayment, the payment is CPR(t)P(t). Therefore, we have the following cashflow model:

E ( X(t) | [illegible](t) ) = [( 1 – CPR(t) – CDR(t)) Y(t) + ( CPR(t) + CDR(t) [illegible] ) ] P(t) (11) where X(t) is the cashflow subject to the uncertain prepayment and default risks. $\Omega(t)$ is the information set available at time t, and $\delta$ is the recovery ratio at default.

But the CPR(t) and CDR(t) depend on the prevailing yield curve level and shape. And therefore, we need an interest rate model to generate the interest rate scenarios. Using equations (7) – (8), we can specify the cash flows of a mortgage loan each month along the interest rate scenario. The present value of this cash flow is then determined using the vector of one month rates discounting the cash flows. The value is called the pathwise value for an interest rate scenario. Interest rate scenarios are determined randomly based on the generalized Ho-Lee model from the average of the 1000 pathwise values. This is the fair value of a mortgage loan, before introducing the option adjusted spread, which we will discuss in a later section.

### *2.3. Data*

The loan level data are from LoanPerformance, a firm that remarkets mortgage servicing data. The loan sample is restricted to the fixed rate fully amortizing 30 year maturity loan product. All of the loans are backed by owner occupied single family residence. The original loan balance spans both the conforming market and the Jumbo market, and credit quality is broad.

## Table 1: Loan Characteristics at Origination by Cohort Year

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| Cohort | | Loans | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|---|
| 1995 Cohort | Loan Amount | 4897 | $ 241,461 | 140809 | $ 10,650 | $ 1,000,000 |
| | Contract Rate | 4897 | 8.41 | 0.87 | 5.50 | 16.25 |
| | Credit Score | 4897 | 693 | 73 | 444 | 823 |
| | Original LTV | 4897 | 78 | 14 | 0 | 119.41 |
| 1996 Cohort | Loan Amount | 16657 | $ 241,632 | 142968 | $ 10,000 | $ 1,600,000 |
| | Contract Rate | 16657 | 8.49 | 0.93 | 4.75 | 15.76 |
| | Credit Score | 16657 | 696 | 64 | 406 | 829 |
| | Original LTV | 16657 | 76 | 13 | 0 | 216 |
| 1997 Cohort | Loan Amount | 44123 | $ 243,585 | 135439 | $ 11,700 | $ 1,500,000 |
| | Contract Rate | 44123 | 8.21 | 0.80 | 5.88 | 18.46 |
| | Credit Score | 44123 | 711 | 58 | 440 | 832 |
| | Original LTV | 44123 | 76 | 12 | 0 | 110 |
| 1998 Cohort | Loan Amount | 103258 | $ 270,949 | 147803 | $ 8,800 | $ 2,000,000 |
| | Contract Rate | 103258 | 7.75 | 1.09 | 5.00 | 18.25 |
| | Credit Score | 103258 | 710 | 62 | 400 | 845 |
| | Original LTV | 103258 | 74 | 12 | 0 | 120 |
| 1999 Cohort | Loan Amount | 66992 | $ 265,811 | 155921 | $ 10,000 | $ 2,000,000 |
| | Contract Rate | 66992 | 7.97 | 1.43 | 5.00 | 17.55 |
| | Credit Score | 66992 | 699 | 68 | 400 | 827 |
| | Original LTV | 66992 | 75 | 13 | 0 | 151.26 |
| 2000 Cohort | Loan Amount | 46806 | $ 240,382 | 166451 | $ 10,000 | $ 3,770,000 |
| | Contract Rate | 46806 | 9.27 | 1.46 | 5.25 | 18.00 |
| | Credit Score | 46806 | 685 | 74 | 400 | 829 |
| | Original LTV | 46806 | 75 | 18 | 0 | 180.29 |
| 2001 Cohort | Loan Amount | 66305 | $ 335,845 | 174354 | $ 10,000 | $ 2,105,000 |
| | Contract Rate | 66305 | 7.87 | 1.08 | 5.00 | 15.30 |
| | Credit Score | 66305 | 710 | 60 | 424 | 832 |
| | Original LTV | 66305 | 75 | 14 | 6 | 143.66 |
| 2002 Cohort | Loan Amount | 50927 | $ 309,970 | 185943 | $ 14,450 | $ 1,600,000 |
| | Contract Rate | 50927 | 7.33 | 1.04 | 4.70 | 14.63 |
| | Credit Score | 50927 | 712 | 59 | 436 | 848 |
| | Original LTV | 50927 | 75 | 16 | 7.1 | 120.88 |
| 2003 Cohort | Loan Amount | 55086 | $ 281,427 | 188239 | $ 15,000 | $ 1,800,000 |
| | Contract Rate | 55086 | 6.60 | 1.02 | 3.88 | 12.38 |
| | Credit Score | 55086 | 707 | 59 | 438 | 837 |
| | Original LTV | 55086 | 76 | 17 | 6 | 121.3 |
| 2004 Cohort | Loan Amount | 27694 | $ 244,386 | 182180 | $ 14,725 | $ 1,800,000 |
| | Contract Rate | 27694 | 6.68 | 1.01 | 4.75 | 12.88 |
| | Credit Score | 27694 | 703 | 60 | 425 | 827 |
| | Original LTV | 27694 | 79 | 16 | 9.62 | 116.42 |
| 2005 Cohort | Loan Amount | 30279 | $ 321,684 | 231922 | $ 15,000 | $ 3,000,000 |
| | Contract Rate | 30279 | 6.49 | 0.95 | 1.00 | 13.05 |
| | Credit Score | 30279 | 707 | 62 | 477 | 847 |
| | Original LTV | 30279 | 75 | 15 | 7.9 | 126 |
| 2006 Cohort | Loan Amount | 18905 | $ 365,847 | 260079 | $ 13,600 | $ 3,500,000 |
| | Contract Rate | 18905 | 6.92 | 0.86 | 4.75 | 12.38 |
| | Credit Score | 18905 | 709 | 59 | 482 | 821 |
| | Original LTV | 18905 | 74 | 14 | 8 | 107 |

Table 1 summarizes the loan level data at origination. Overall the sample contains 531,929 loans which were originated between 1995 and 2006. Across the loan cohorts the average original loan size varies between $240,000 and $365,000 - this variation is partly due to the mix of jumbo/non-jumbo loans in the cohort. Interestingly, the average original LTV across the cohorts is fairly constant at 75%. The contract rates are not annual percentage rates (APR) and must be viewed carefully. In particular, the contract rate on any particular loan may have been "bought" down through the purchase of points at origination. During model estimation, we would ideally include both points paid and contract rate, yet in the absence of APR we excluded both for estimation to avoid measurement error associated with the contact rate. This issue was covered in the section focusing on the construction of the refinance spread variable. Please note that the contact rate is used during our valuation stage. The average cohort credit scores are tightly distributed between 685 and 712; however, within any particular cohort, the credit score ranges between 400 and the low 800s. To this extent, the sample covers a wide spectrum of borrower credit quality.

There are more than 15 million loan-month observations. In terms of original unpaid principal balance, the sum of the original loan balances is approximately $150 billion. Over the reporting period, we observe 410,399 prepayments and 21,982 defaults. The remaining observations represent scheduled payments. Table 2 offers a closer view of the cohort performance. The table identifies peak defaults at 9% for the year 2000 loan cohort. The refinance boom in 1998 is seen with the great volume of originations in our sample. Interestingly, 95% of these 1998 originated loans prepaid by July 2006.

**Table 2: Termination Events by Cohort**

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| Cohort | $Volume (millions) | Prepaid (loans) | Defaulted( loans) | Total Terminations | Current @ 7/2006 |
|---|---|---|---|---|---|
| 1995 | $ 1,182 | 4,455 | 271 | 4,726 | 171 |
| 1996 | $ 4,025 | 15,589 | 829 | 16,418 | 239 |
| 1997 | $ 1,075 | 42,605 | 1,223 | 43,828 | 295 |
| 1998 | $ 27,978 | 98,685 | 3,618 | 102,303 | 955 |
| 1999 | $ 17,807 | 61,787 | 3,969 | 65,756 | 1,239 |
| 2000 | $ 11,251 | 41,672 | 4,294 | 45,966 | 840 |
| 2001 | $ 22,268 | 61,985 | 2,590 | 64,575 | 1,730 |
| 2002 | $ 15,786 | 43,089 | 1,991 | 45,080 | 5,847 |
| 2003 | $ 15,503 | 27,848 | 1,902 | 29,750 | 26,336 |
| 2004 | $ 6,768 | 9,780 | 886 | 10,666 | 17,028 |
| 2005 | $ 9,740 | 3,536 | 381 | 3,917 | 26,362 |
| 2006 | $ 6,916 | 368 | 28 | 396 | 18,509 |

Table 2 offers a closer view of the cohort performance. The table identifies peak defaults at 9% for the year 2000 loan cohort. The refinance boom in 1998 is seen with the great volume of originations in our sample. Interestingly, 95% of these 1998 originated loans prepaid by July 2006. Prepaid loans are identified in the transaction data as loans with a zero unpaid principal balance prior to maturity, and have never been in the state of 90 days delinquent. The second criterion is required to avoid mislabeling "work outs" as prepayments when they actually should be classified as defaults. Also critical to the data design and estimation effort is the definition of the date of prepayment and default. For prepayments, the date in which the balance transitions to zero is the date of the prepayment. In the case of defaults, the date of the last completed full payment is labeled the default date.

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The market data is based on June 1, 2006 data. The Treasury curve is based on Federal Reserve Bank data. Swaption prices used to determine the implied volatility of the yield curve movement are obtained from Bloomberg Financial Services.

## 3. Prepayment/Default Model Specifications & Empirical Results

The default and prepayment model parameters are reported in Appendix C. Appendix C includes four alternative default and prepayment model specifications; (1) the age spline only; (2) the age spline plus cohort and transaction month variables; (3) the full model less credit scores and unpaid principal balance spline; and (4) the full model. Across the four models marginal improvements are seen, particularly within the default equation. The parameter estimates for the default equation tend to settle down when the credit scores and original balances are included in the model. The following subsections consider each group of parameter estimates in turn.

### *3.1. Seasoning effects*

"Seasoning" with regard to the loans describes the prepayment and default rate as a function of the age (in months) of the loans. The top panel in Appendix C lists the age spline parameter estimates with standard errors reported under each parameter estimate. Across the four reported models, it is noticeable that the parameter estimates for model 1 are dissimilar to the other models. In particular, notice that the different magnitude and direction of several of the parameters. For example, the first two age parameters are four times larger as well as the opposite sign than the other models. This suggests that modeling defaults and prepayments purely based on historical outcomes in the absence of controlling for borrower, loan and economic characteristics could lead to mispricing. The inclusion of these control variables in the other models appears to stabilize the seasoning parameters.

All of the prepayment seasoning parameters are statistically different than zero. However, the default seasoning parameters lack the same level of precision as the prepayment model. The later months in the times series are supported by fewer and fewer observations. As such, fitting the seasoning parameters in the later periods is challenging. Otherwise, the spline parameters are rather robust to the alternative specifications considered in Appendix C.

Interpreting the parameter estimates in Appendix C is not straight forward. The spline parameters are cumulative in design and the Multinomial Logit structure requires the transformation depicted in equation 3a for interpretation. To this extent, Figure 1 plots the expected monthly conditional prepayment rates (equation 3a) over loan age in months against the actual prepayment rates.

**Figure 1: Conditional Prepayment Rates**

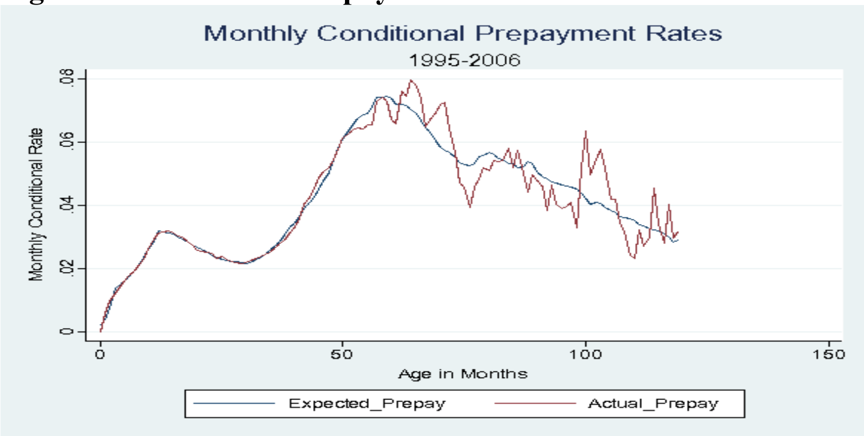
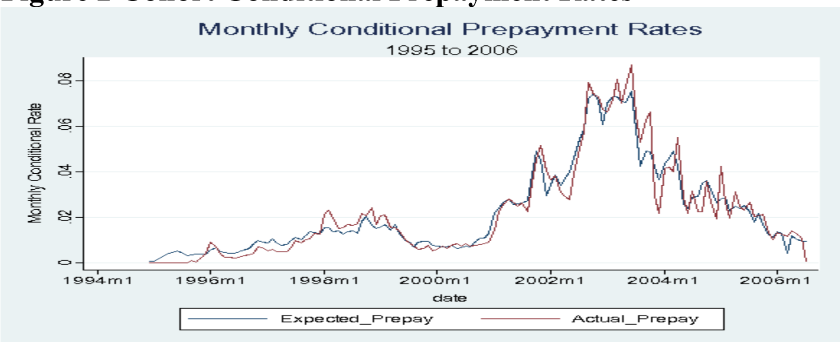



Figure 1 demonstrates the ability of the age spline function to capture the nonlinearity of the aging prepayment process[2]. Although the model utilizes 12 spline knots, there are three distinct segments depicted in the prepayment plot; a ramping up of prepayments during the first 13 months, a gradual decline in prepayments over the next year and a final ramping up. Although the chart only spans the first 120 months of the contracted term, the model can forecast out to the full maturity, 360 months.

The second way to view the model accuracy is the plot actual and expected rates across multiple cohorts through calendar time. Figure 2 demonstrates in sample robustness across all loan cohorts and through time.

**Figure 2 Cohort Conditional Prepayment Rates**




[2] The expected conditional prepayment rate is constructed within sample for the 15 million loan months. The figure reports the mean expected conditional prepayment rate calculated at each month using model (4). Values of the independent variables are the same as those used in estimation, as such this is an "in sample" forecast. The actual prepayment rate is simply defined as the ratio of the number of prepayments to the number of at risk loans in each month. The loan sample is described in table 1 and the appendix tables.

Figure 3, plots the expected conditional default rates against the actual default rates. In comparison to the conditional prepayment rates, the default rates are ten times lower that the monthly prepayment rates. For example, at the 20th month, the expected conditional monthly default rate is about 0.25% while the conditional monthly prepayment rate is approximately 2.5%.

**Figure 3: Conditional Default Rates**

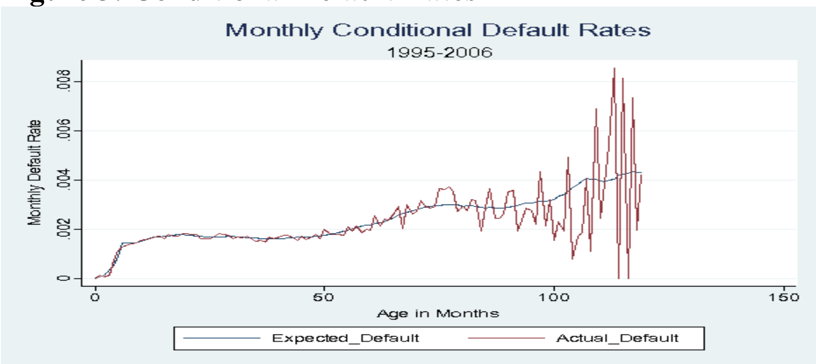
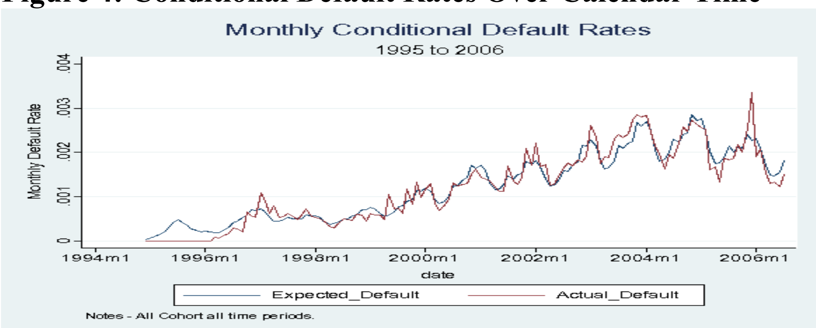



Figure 3 also demonstrates the impact of the thinning loan sample in the later months. Although one loan lived 139 months, the majority of the loan sample terminates before the 100th month. For example, at month 100 there are only 1936 loans at risk of terminating, and these loans must have been originated prior to March of 1998. Beyond month 75 the absence of loans in the sample leads to large swings in the actual default rate. And after the 90th month, fewer than 10 loans default per-month in the sample, and in figure 4 where the actual default rate touches zero, a single loan defaulted . Although the forecasting power of the default model appears lacking in the later years, figure 4 suggests that model is rather reasonable. Figure 4 plots expected and actual default rates for all loans in the sample over calendar.

**Figure 4: Conditional Default Rates Over Calendar Time**




### *3.2. Seasonality*

The estimates of the seasonality coefficients are presented in Table 3. January is the excluded month, as such, the parameter estimates in the model are relative to January. In the full model, both the conditional default rate and the conditional prepayment rate are related to the calendar months, but in a different way. Statistically, the seasonality parameters in the prepayment function are significantly different than zero. The parameter for September is relatively smaller than all of the other months - suggesting fewer prepayments in September. The most of the seasonality parameters are significantly different than zero. The exception is February.

The model suggests that loans are more likely to default (relative to January) during the month of March, while least likely to default in May. Meanwhile, September is the slowest month of prepayment according to the parameter estimates.

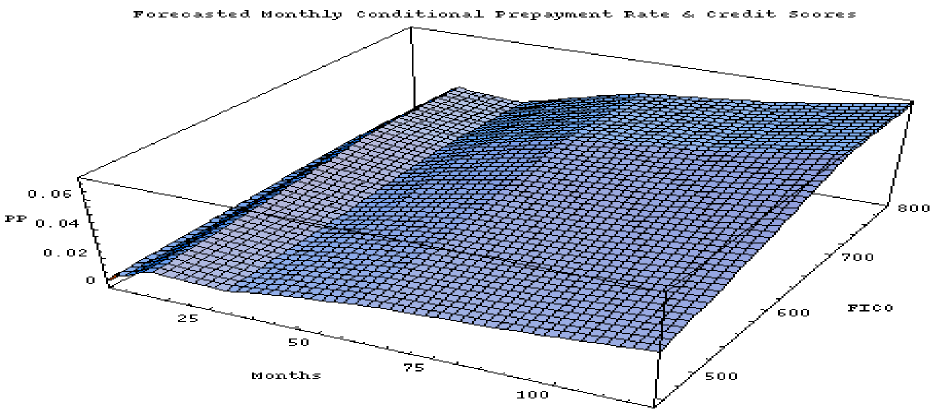
### *3.3. Cohort Effect*

Both default and prepayment experiences may be affected by factors not considered in the model. For example, such factors may include the macroeconomics variables like consumer income and confidence. In addition, local economic conditions driving housing prices are rarely directly observed. These factors are known to be time varying as well as known to impact the default and prepayment experience. In order to capture these factors without an explicit model of them, we use the cohort indicators defined by the origination year. In addition, omitted structural changes in the primary mortgage are captured by the cohort variables. Statistically, all of the cohort parameters are significantly different than zero. Please visit Appendix C, Table 3 for the full set of parameter estimates.

### *3.4. FICO Effect*

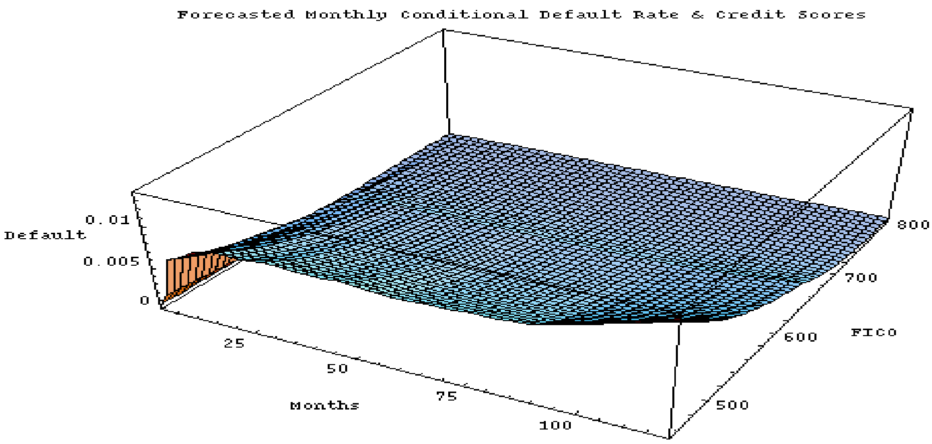
The credit scores deployed in the model are FICO scores reported at loan origination. FICO scores enter into the model in the form of a spline function with three knot points. Figure 5 demonstrates the sensitivity of the conditional prepayment rate to different in credit scores. As suggested by the model, higher credit scores lead to greater prepayment speeds. The figure also suggests that the difference in prepayment speeds increases as the loan ages. For example, while a high credit score mortgagor (who has a high credit rating) tends to prepay faster, the prepay speeds also increases with age. To this extent, the plane covering months zero to 25 appears rather similar across the FICO scores. However, following the first 25 months, the conditional prepayment rates grow faster with increasing credit scores.

**Figure 5. Conditional Prepayment Rate and Credit Scores**




In the default model, the credit score spline is a very explanatory set of parameters. The conditional default rate increases rapidly as the credit score declines below 650. In Figure 6, the conditional default rate for a loan with an 800 FICO appears negligible over the first 100 months. However, the identical loan with a FICO of 600 faces a much steeper conditional default function. Another interesting finding suggested by the figure is that loans with lower credit scores face higher expected conditional default rates earlier in their lives relative to higher score borrowers.

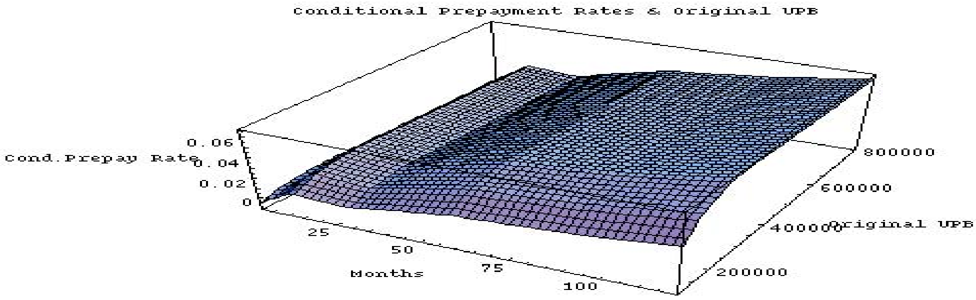
**Figure 6. Conditional Default Rates and Credit Scores**

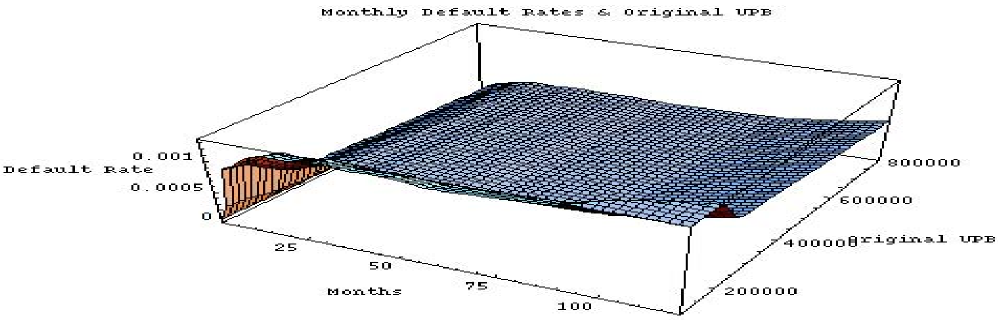



### *3.5. Size Effect: Hot and Cold Money*

The coefficients of the linear spline model of the original loan size effect are presented in Table 3. Similar to the age spline used in the model, this spline is cumulative in design, such that large loans utilize more spline knots than small loans in the default and prepayment calculations. In both the default and prepayment equations, all of the spline parameters are statistically significant from zero. The figure 7 suggests that the prepayment rate increases with the original loan size, though the rate is highest around $250,000. This result suggests that the larger the outstanding amount, the *hotter* is the money. Conversely, for the default model, the default rate decreases with the increase in the loan size. Figure 7 depicts this behavior.

**Figure 7. The Size Effect on the CPR and CDR**




This is the last effect that applies to both prepayments and defaults. The remaining two effects are related only to prepayments.

### *3.6. Yield Curve Slope*

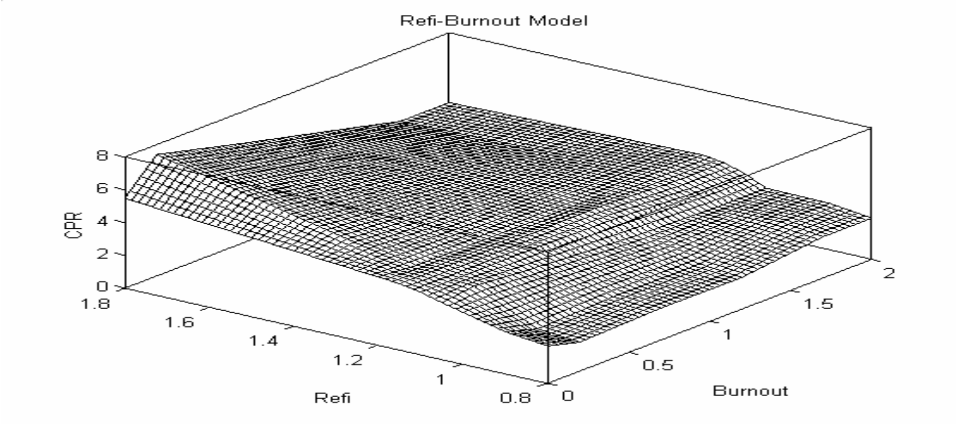
The slope of the yield curve is defined as the 10 year Treasury rate (in percent) net of the 2 year rate (in percent). The slope is used for the conditional prepayment rate model and not for the conditional default rate model, as the default tends not to be dependent on the shape of the yield curve. We expect that the steeper yield curve could lead to higher prepayments out of the 30yr FRM. For example, the relative cost to short termed mortgage products could tilt borrowers toward short termed maturities.

The estimates suggest that a steeper yield curve generates faster prepayment speeds. Interestingly, an inverted yield curve, where the spread is negative, leads to relatively slower prepayment speeds.

### *3.7. Refinance Ratio and the Burnout Effect*

The refinance-burnout function is depicted in Figure 8. The figure plots the plane(s) between refinance ratio, burnout function and the monthly conditional prepayment rate. As the refinance ratio increase from unity, the mortgage becomes more in the money to refinance. The burnout function captures the number forgone refinance basis points above a threshold. Interestingly, some borrowers tend to prepay less at burnout levels less than 0.25. This suggests less than ruthless refinancing. For example, when burnout equals zero, the prepayment rates for refinance ratios greater than 1.2 are less than those with burnout greater than 0.25. The refinance behavior is consistent with the shape of the S curve at different levels of the burnout. That is, the results are consistent with the standard result that the refinance rate increases with a fall in the current mortgage rate.

**Figure 8: Refinance Ratio and Burnout**




Higher refinancing is observed in light of burn out, though is not the case when the rate ratio is less than 1.10. There are fewer observations for the burnout factor higher than 1.7, as such, the lack of data may lead to refinancing rate estimates higher than expected.

### *3.8. Default and Prepayment Model Summary*

The results suggest that the interactions between default and prepayment risks are complex. Specifically, the very low and very high FICO score slow the prepayment rate yet higher FICO scores always reduce the default rate. Both the seasonality and seasoning have impacts on both the default and the prepayment rates but the functional forms with respect to time are different. Meanwhile, the loan size is nonlinearly related to both default and prepayment rates. Increasing loan size reduces default rates at all levels yet increases prepayments until the loan is very large. These results suggest that using a simple reduced form default model to determine the "mortality" of a mortgage loan is inappropriate, because such a model assumes that we can isolate the "event of a death of a mortgage" from other effects, something that our empirical results fail to support.

## 4. Valuation of the Mortgages

This section reviews the calculation of the option adjusted spread given the estimated default and prepayment equations and applies the OAS to further investigate both the credit and interest rate behavior of a sample of loans. More specifically, section 4.1 discusses the valuation methodology, and section 4.2 reports the OAS values across alternative recovery rate assumptions. The last two sections of the paper consider direct applications of our model, namely, estimating the cost of guaranteeing mortgage loans and measuring the credit impact on duration.

### *4.1. Determining the OAS*

We now proceed to use the estimated prepayment-default model to determine the valuation of a mortgage loan at origination. The loan data is based on originations during the month of July in 2006. In order to control various factors in determining the contract rate, we selected 240 loans that have a contract rate of 6.285%. In our research, we have repeated the tests without selecting a sample of a specific interest rate. The main results do not change.

At origination, the bank pays the par amount for the mortgage. When a mortgage prepays, we assume that the bank receives the remaining principal. However, when a mortgage defaults, the bank receives the remaining principal times a constant recovery ratio. A constant spread or option adjusted spread (OAS) is added to the one month Treasury rate along a scenario. These are the monthly discount rates to determine the pathwise value of the mortgage loan. Following the standard methodology in determining the option adjusted spread (OAS) of a mortgage loan, we determine the OAS to be the constant spread such that the mean of the pathwise values of the mortgage loan equals par. The OAS can be interpreted as the gross profit of funding a mortgage loan. It is the interest income net of the combined prepayment and default options sold to the mortgagors.

All the independent variables in the default-prepayment equations are observable at the issuance date for each mortgage loan, except for the cohort variable. The coefficient of the cohort variable is assumed to be the average of the past estimates. We use the Treasury yield curve observed on the origination date to value each mortgage. While the term structure of volatilities does affect the embedded options in the mortgages, it does not affect the value of the mortgage value significantly.

### *4.2. OAS Analysis*

The OAS is estimated over a range of recovery ratios: 80%, 85%, 90%, 95% and 100%. The results are presented in Table 3.

**Table 3 OAS Analysis of the Mortgage Loans interest rate 6.285**

| | |
|---|---|
| | -----------------------**recovery ratio**-------------------- |

| | Principal | FICO | LTV | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|---|---|---|
| Mean | $311,878 | 715.621 | 73.9163 | 0.00713 | 0.00809 | 0.00905 | 0.01000 | 0.01094 |
| Std Dev | $228,665 | 44.5498 | 12.8826 | 0.00238 | 0.00168 | 0.00100 | 0.00035 | 0.00042 |
| Var Coef | 0.73319 | 0.06225 | 0.17429 | 0.33342 | 0.20796 | 0.11050 | 0.03505 | 0.03799 |
| Correl (Principal) | | | | 0.41329 | 0.43652 | 0.48893 | 0.69935 | 0.00782 |
| Correl (FICO) | | | | 0.84677 | 0.83627 | 0.80802 | 0.61151 | -0.89904 |
| Correl (LTV) | | | | -0.16427 | -0.16599 | -0.16914 | -0.17046 | 0.11742 |

The average size of the principal amount of the sample is $311,878, and the average FICO score is 715. The variation of the principal amount is quite significant, with a standard deviation of $228,665, or a coefficient of variation of 73%. The average OAS assuming a 100% recovery ratio, the case for mortgage guaranteed loans, is 1%. This result is consistent with that obtained by Longstaff (2002), where the spread is estimated for mortgage-backed securities. However, when the principal is not guaranteed, the spread would be tightened, since part of the principal would be deducted as a loss in default. Specifically, the result shows that the OAS tightens by approximately by 38 basis points when the recovery ratio lowers to 80%.

The variations of the OAS across the mortgages are significant. The loans' OAS for the range of recovery ratios are depicted in Table 4

**Table 4 Option Adjusted Spreads and the FICO Relationship**

OAS (basis points)

| | Recovery Ratio | | | | |
|---|---|---|---|---|---|
| **FICO** | **80** | **85** | **90** | **95** | **100** |
| 550 | -38 | 6 | 49 | 91 | 132 |
| 600 | 5 | 36 | 66 | 97 | 127 |
| 650 | 45 | 64 | 83 | 102 | 121 |
| 700 | 82 | 90 | 98 | 106 | 114 |
| 750 | 97 | 101 | 104 | 108 | 111 |
| 800 | 104 | 105 | 107 | 108 | 110 |
| 850 | 107 | 107 | 108 | 108 | 109 |

Table 4 shows that, as expected, the variations of the OAS across different recovery ratios are minimal when the FICO scores are high, yet this is not the case when in light of greater default risk; when the FICO scores are lower (on the left hand side.). However, the variation of OAS is high across the loans. Part of the variation is explained by the variation of the FICO scores and size, referring to Table 4 where we report the significant correlations of the FICO scores and the size to the OAS of the loans.

As expected, for low recovery ratios, the correlation between the FICO scores and the OAS is positive (loans with lower FICO scores have a lower option adjusted spread). The result shows that when the recovery ratio is between 90% and 95%, the correlation is near zero, and that the default adjusted spread (or profitability) is not biased for or against the default risk of the loans.

Consider the case of 100% recovery ratio, with the presence of transaction costs, the OAS tends to be higher for smaller principal mortgage loans. For this reason, the correlation of the variation of the principal size and the OAS is negative ( -39%). However, when the recovery ratio decreases, and since smaller mortgage loans tend to have a higher default rates, the OAS tends to tighten. Neither the variations of the mortgage loan size nor the FICO scores can explain the variations of the OAS when the recovery ratio is 90%. This result suggests that the loan origination profitability varies significantly depending on the servicer's ability (or bank's ability) to recover defaulted loans.

### *4.3. The Cost of Guarantee*

Our results can be extended to study the appropriate charges in guaranteeing default losses. If the mortgage loan is guaranteed, then the investor would receive the par amount in the event of default. Hence, the investor can assume that the recovery ratio is 100%. The widening of the OAS with the guarantee is therefore the expected cost to the guarantor. In particular, given a recovery ratio, *R*, we define the guaranteed premium *P* to be

$$P = \text{OAS (recovery ratio = 100\%)} - \text{OAS (recovery ratio} = R) \qquad (12)$$

The summary results are presented Table 5.

**Table 5: Guaranteed Premiums & Recovery Ratios**

| | Descriptive Stats on the Guaranteed Premium | | | |
|---|---|---|---|---|
| Recovery Ratio | **80** | **85** | **90** | **95** |
| Mean Premium | 0.381% | 0.285% | 0.189% | 0.094% |
| St-Dev | 0.275% | 0.205% | 0.136% | 0.068% |

The result shows that the average premium for each recovery ratio assumed, ranging from 10 basis points to 38 basis points. However, the guaranteed premium should be dependent on the FICO score and the principal size, for a given recovery ratio. The model of such a relationship is presented below.

$$\ln P = \alpha + \beta_{OUPB} OUPB + \beta_{FICO} \ln FICO + \beta_{OLTV} \ln OLTV \qquad (13)$$

Where $\beta_{OUPB}$ and $\beta_{FICO}$ are the elasticities of the guaranteed premium to the principal size and FICO score respectively.

**Table 6 Guaranteed Premium Model**

| **Coefficient** | **ratio 80** | **ratio 85** | **ratio 90** | **ratio 95** |
|---|---|---|---|---|
| Intercept | 75.8095 | 75.3908 | 74.8573 | 74.039 |
| Principal(OUPB) | -0.4459 | -0.4455 | -0.445 | -0.4445 |
| FICO | -11.8947 | -11.8757 | -11.8572 | -11.8391 |
| OLTV | 0.4725 | 0.4720 | 0.4716 | 0.4711 |

| | | | | |
|---|---|---|---|---|
| ln(intercept) | 4.3282 | 4.3227 | 4.3156 | 4.3046 |
| | | | | |
| t statistics | | | | |
| Intercept | 92.259 | 91.780 | 91.162 | 90.198 |
| Principal(OUPB) | -41.193 | -41.163 | -41.134 | -41.106 |
| FICO | -99.064 | -98.939 | -98.819 | -98.703 |
| OLTV | 14.971 | 14.961 | 14.952 | 14.943 |
| Adj R Square | 0.96409 | 0.96405 | 0.96402 | 0.96398 |

Table 6 suggests that the model of the guaranteed premium has significant explanatory power, with adjusted R square in excess of 97% in all cases. Further, the coefficients have *t* statistics which are significantly different than zero, and are stable over the range of recovery ratios. The elasticities are -0.44, -11.89 and 0.48 for the original loan amount, FICO score, and LTV respectively. This result shows that the principal amount is less important in determining the guaranteed premium and 1% increase in the FICO score would lead to 11.89% decrease in the guaranteed premium for the given loan sample.

### *4.4. Impact of the Credit Risk on Duration*

The credit risk has confounding effects on both the timing and size of mortgage cash flows. For example, for borrowers with very low credit scores (high credit risk), the results show that these mortgagors tend to prepay more slowly than more credit worthy borrowers. On the other hand, the same borrower is more likely to default. The net risk to the investor of receiving their principal back too soon is analytically indeterminant, yet empirically measurable. The duration, as defined to be the price sensitivity to a parallel shift of the yield curve, of a mortgage intersects these two opposing factors of the mortgagor's put option on the credit risk and call option on the interest rate risk into account.

**Figure 9. Duration, FICO and OLTV with 100% Recovery Rates**

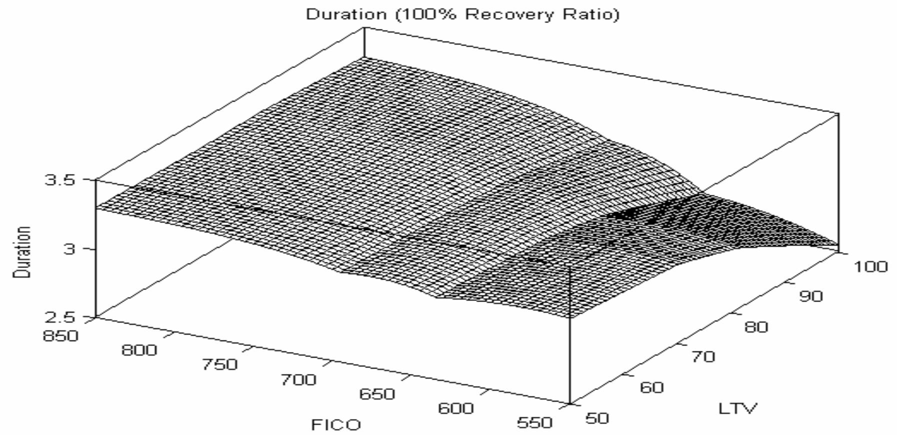


Figure 9 depicts the duration a function of the FICO Score and the original LTV for fully guaranteed loans (i.e. 100% recovery ratio), and it shows that the default effect dominates for borrowers with low FICO scores and high original LTVs. The figure suggests that higher FICO scores and lower LTVs lead to longer duration. However, such behavior is changed when the loan is not guaranteed or is assumed to have a recovery rate of less than 100%

**Figure 10. Net Effect of Alternative Recovery Rates on Duration**

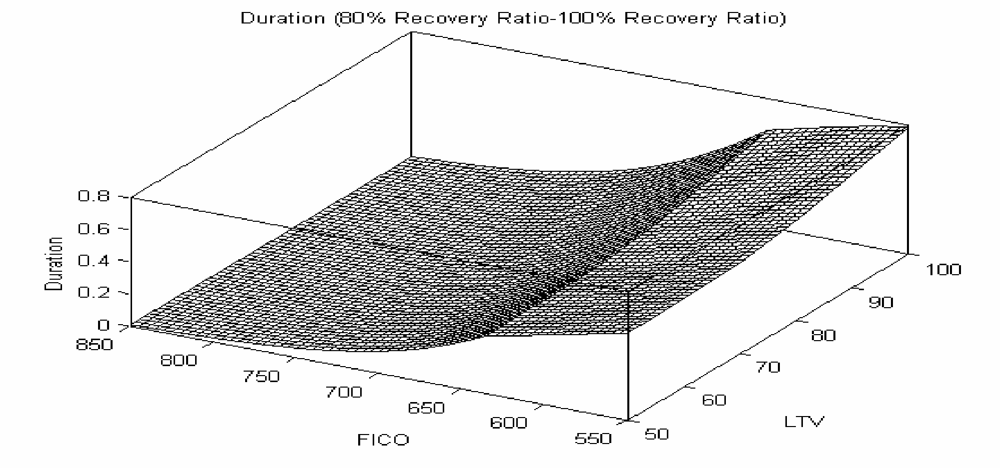



Figure 10 offers a more direct view of the impact of alternative recovery rate assumptions on duration across the credit spectrum. This figure is the point by point difference between the duration assuming 100% recovery rate, and that assuming 80% recover rate. The interpretation of Figure 10 is straightforward; areas with zero values indicate no difference between duration calculated at 100% and 80% recovery rates. Positive points on the graphic indicate increased duration when we decrease the recovery rate by 20 percentage points. For loans with high credit scores, we see the difference between the recovery rate assumptions as negligible. Yet as we move down the FICO score axis in the duration increases for all valuates of original LTV. The recovery rate assumptions are most important when working with low credit quality loans. As expected, the greatest difference in duration is the combination of high original LTV loans and low credit score borrowers. These results show the importance of taking the credit risk into consideration even for the guaranteed loans, because, the confounding effects of the put and call options in the mortgage loans can have conflicting impact on the loan duration.

## 5. Conclusion

Mortgage loan valuation inclusive of both the credit and interest rate risk is critically important to all market participants; investment bankers, investors, originators, servicers and regulators. In addition to the obvious need for accurate valuation models, changes in accounting rules provide some of the impetus in enhancing the state of mortgage valuation models. The introduction of fair value accounting, fair disclosure rule, IAS

39/FASB 133 (fair value hedge accounting), FASB 140 (fair valuation of mortgage servicing) and FASB 115 (fair valuation of assets) are some examples. The increase use of risk management techniques and disclosures is another factor. For example, the Basel II Pillar I and Pillar II for capital requirements and the use of Value-at-Risk (VaR) to identify the potential risks of the portfolio result in banks determining the economic value of the mortgage loans. Beyond satisfying the regulatory needs, the best practice of asset liability management and profitability measure of a bank often requires the use of fair valuation of these mortgage loans. Therefore, banks find broad applications of such valuation models in their financial disclosures, profitability and risk management.

Valuation of mortgage loans is also important for non-bank institutions. Investors of whole loans have to evaluate the underlying loan value. Government sponsored enterprises, and other credit guarantors of mortgage loans (such as, FHA) need similar models to determine the cost of the guarantee of the credit risks net of fee income. The mortgage insurance business requires consideration of default probabilities in regard to the costs and prepayment likelihoods in regard to the stream of fee income. Mortgage servicers use the fair valuation of the mortgage loans to hedge their servicing fees. Finally, investment banking uses these models to design financial innovations (for example credit derivatives, structured products), and hedging strategies.

This paper provides a model of the valuation of mortgage loans on the balance sheets of the banks. Many banks have originated and held or bought mortgage loans from external sources. They constitute a significant part of their balance sheet. Many of the mortgages are neither securitized nor passing the default risk to the mortgage guarantors.

This paper has estimated the prepayment-default model and presented a valuation model of a mortgage loan. The result identifies the competing effects of default and prepayment on the valuation of a mortgage loan. Further, the valuation model shows that the OAS of the mortgage loan is sensitive to the assumed recovery ratio. The paper presents a model of the premium in guaranteeing a mortgage loan and studies the impact of credit risk on the mortgage duration.

The methodology presented in the paper can be extended to value other mortgage loans, including the 15 year fixed rate, balloon, and interest-only mortgages. The valuation model can have a broad range of analytical applications including determining the key rate durations of the mortgages for hedging, and the model can be used to value whole loans, mortgage servicing fees, and the cost of default guarantees

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## Appendix A: Refinance-Burnout Function

Let δ(refinance_spread > k) is the delta function which equals 1 if the condition is satisfied, otherwise zero. Let $burn\_out_j$ ($j = 1,...,5$) be defined below:

$burn\_out_{k,i,t=1} = \delta\ (L(k) =< burn\_out_{i,t} < L(k+1))$ for k =1,…,5

| k | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| L(k) | 0 | 0.2 | 0.7 | 1.2 | 1.7 | 10 |

We now define the independent variables to be:

$yt\_j\_h = (\delta\ (L^*(h) < refinance\_spread_{i,t} < L^*(h+1))^* \ (refinance\_spread_{i,t} - L^*(h)) + \delta\ (refinance\_spread_{i,t} >= L^*(h+1))^*(L^*(h+1) - L^*(h))^* burn\_out_{j,i,t}$

$yt\_j\_5 = \delta\ (1.2 < refinance\_spread_{i,t})^*(refinance\_spread_{i,t} - 1.2)^* burn\_out_{j,i,t}$

j = 1, …,5   h = 0, …, 5

| h | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| L*(h) | 0 | 0.9 | 1.05 | 1.10 | 1.15 | 1.2 |

## Appendix B: The Two Factor Generalized Ho-Lee Model

Let $P_{i,j}^{n}(T)$ be the price of a $T$ year bond at time $n$, at state $(i, j)$. Then the bond price is specified by combining two one-factor models. Specifically, we have

$$P_{i,j}^{n}(T)=\frac{P(n+T)}{P(n)}\prod_{k=1}^{n}\left(\frac{1+\delta_{0,1}^{k-1}(n-k)}{1+\delta_{0,1}^{k-1}(n-k+T)}\right)\prod_{k=1}^{n}\left(\frac{1+\delta_{0,2}^{k-1}(n-k)}{1+\delta_{0,2}^{k-1}(n-k+T)}\right)\prod_{k=0}^{i-1}\delta_{k,1}^{n-1}(T)\prod_{k=0}^{j-1}\delta_{k,2}^{n-1}(T) \tag{A.1}$$

where

$$\begin{aligned}\delta_{i,1}^{n}(T)&=\delta_{i,1}^{n}\delta_{i,1}^{n+1}(T-1)\left(\frac{1+\delta_{i+1,1}^{n+1}(T-1)}{1+\delta_{i,1}^{n+1}(T-1)}\right)\\ \delta_{i,2}^{n}(T)&=\delta_{i,2}^{n}\delta_{i,2}^{n+1}(T-1)\left(\frac{1+\delta_{i+1,2}^{n+1}(T-1)}{1+\delta_{i,2}^{n+1}(T-1)}\right)\end{aligned} \tag{A.2}$$

and the one period forward volatilities are given by definition,

$$\begin{aligned}\delta_{i,1}^{m}(1)&=\delta_{i,1}^{m}=\exp\left(-2\cdot\sigma_{1}(m)\min\left(R_{i,1}^{m},R\right)\Delta t^{3/2}\right)\\ \delta_{i,2}^{m}(1)&=\delta_{i,2}^{m}=\exp\left(-2\cdot\sigma_{2}(m)\min\left(R_{i,2}^{m},R\right)\Delta t^{3/2}\right)\end{aligned} \tag{A.3}$$

where the functions $\sigma_{j}(n)=(a+bn)\exp\left(-cn\right)+d$ is specified by the parameters $a$, $b$, $c$, and $d$, which can be obtained from the calibration to the market price of swaption.

Using the direct extension, we can specify the one period rates for the two-factor model for any future period $m$ and state $i$, and $R_{i,1}^{m}$ and $R_{i,2}^{m}$ are defined by

$$\begin{aligned}R_{i,1}^{m}\Delta t&=-\log\left(\frac{P(m+1)}{P(m)}\right)-\sum_{k=1}^{m}\log\left(\frac{1+\delta_{0,1}^{k-1}(m-k)}{1+\delta_{0,1}^{k-1}(m-k+1)}\right)-\sum_{k=0}^{i-1}\log\left(\delta_{k,1}^{m-1}(1)\right)\\ R_{i,2}^{m}\Delta t&=-\log\left(\frac{P(m+1)}{P(m)}\right)-\sum_{k=1}^{m}\log\left(\frac{1+\delta_{0,2}^{k-1}(m-k)}{1+\delta_{0,2}^{k-1}(m-k+1)}\right)-\sum_{k=0}^{i-1}\log\left(\delta_{k,2}^{m-1}(1)\right)\end{aligned}. \tag{A.4}$$

# Appendix C Multinomial Logit Model

**Table 3: Multinomial Logit Model Estimates of Defualt & Prepayment**

(Standard Errors Under Estimates)

| | Model (1) | | Model (2) | | Model (3) | | Model (4) | |
|---|---|---|---|---|---|---|---|---|
| | **Age Spline only** | | **Age Spline, Cohort & Transaction Month** | | **Full Model Less UPB & Refi/Burnout** | | **Full Model** | |
| Log Likelihood | -2641381.9 | | -1.95E+06 | | -1894401.8 | | -1860570.9 | |
| LR ChiSquared | 8.69E+06 | | 3.01E+07 | | 3.02E+07 | | 3.03E+07 | |
| ChiSquared d.o.f | 24 | | 70 | | 83 | | 120 | |
| Pseud R2 | 0.8445 | | 0.8853 | | 0.8885 | | 0.8905 | |
| **Seasoning Spline Variables** | **Prepay** | **Default** | **Prepay** | **Default** | **Prepay** | **Default** | **Prepay** | **Default** |
| age1 [0 < Months < 3) | -2.486 | -5.705 | 0.641 | 0.876 | 0.630 | 0.870 | 0.630 | 1.215 |
| | 0.004 | 0.024 | 0.008 | 0.064 | 0.008 | 0.064 | 0.008 | 0.060 |
| age2 (3=< Months < 6) | 1.261 | 3.640 | 0.080 | 0.504 | 0.070 | 0.499 | 0.070 | 0.531 |
| | 0.005 | 0.026 | 0.005 | 0.022 | 0.005 | 0.023 | 0.004 | 0.015 |
| age3 (6=< Months <9) | -0.045 | -0.134 | 0.104 | 0.005 | 0.101 | 0.002 | 0.106 | -0.003 |
| | 0.004 | 0.015 | 0.004 | 0.015 | 0.004 | 0.015 | 0.004 | 0.010 |
| age4 (9=< Months < 12) | 0.141 | 0.077 | 0.117 | 0.057 | 0.114 | 0.055 | 0.114 | 0.037 |
| | 0.003 | 0.013 | 0.003 | 0.013 | 0.003 | 0.013 | 0.003 | 0.009 |
| age5 (12 =< Months < 18) | -0.022 | 0.011 | -0.007 | 0.027 | 0.000 | 0.018 | -0.015 | 0.014 |
| | 0.002 | 0.006 | 0.002 | 0.006 | 0.002 | 0.006 | 0.001 | 0.004 |
| age6 (18 =< Months < 24) | -0.034 | -0.009 | -0.023 | 0.005 | -0.016 | -0.001 | 0.002 | -0.006 |
| | 0.002 | 0.006 | 0.002 | 0.006 | 0.002 | 0.006 | 0.001 | 0.004 |
| age7 (24 =< Months < 30) | -0.020 | 0.000 | -0.012 | 0.013 | -0.021 | 0.009 | -0.008 | 0.004 |
| | 0.002 | 0.006 | 0.002 | 0.006 | 0.002 | 0.006 | 0.002 | 0.004 |
| age8 (30 =< Months < 40) | 0.049 | -0.005 | 0.053 | 0.007 | 0.022 | 0.000 | 0.028 | -0.003 |
| | 0.001 | 0.004 | 0.001 | 0.004 | 0.001 | 0.004 | 0.001 | 0.003 |
| age9 (40 =< Months < 50) | 0.064 | 0.011 | 0.072 | 0.024 | 0.048 | 0.004 | 0.017 | -0.003 |
| | 0.001 | 0.005 | 0.001 | 0.005 | 0.001 | 0.005 | 0.001 | 0.003 |
| age10 (50 =< Months < 60) | 0.019 | 0.030 | 0.027 | 0.028 | 0.019 | 0.000 | 0.018 | -0.008 |
| | 0.001 | 0.005 | 0.001 | 0.006 | 0.001 | 0.006 | 0.001 | 0.004 |
| age11 (60 =< Months < 90) | -0.017 | 0.010 | -0.008 | 0.022 | -0.002 | 0.004 | 0.011 | 0.004 |
| | 0.001 | 0.003 | 0.001 | 0.003 | 0.001 | 0.003 | 0.001 | 0.002 |
| age12 (90 =< Months ) | -0.015 | 0.007 | -0.004 | 0.014 | 0.014 | 0.014 | -0.002 | 0.016 |
| | 0.002 | 0.006 | 0.002 | 0.006 | 0.002 | 0.006 | 0.002 | 0.005 |
| **Transaction Month** | | | | | | | | |
| February | | | 0.165 | 0.035 | 0.138 | 0.037 | 0.107 | 0.013 |
| | | | 0.008 | 0.031 | 0.008 | 0.031 | 0.007 | 0.021 |
| March | | | 0.203 | -0.061 | 0.234 | -0.058 | 0.165 | -0.104 |
| | | | 0.008 | 0.031 | 0.008 | 0.031 | 0.007 | 0.021 |
| April | | | 0.122 | -0.267 | 0.237 | -0.262 | 0.168 | -0.333 |
| | | | 0.008 | 0.033 | 0.008 | 0.033 | 0.007 | 0.023 |
| May | | | 0.147 | -0.403 | 0.196 | -0.396 | 0.218 | -0.452 |
| | | | 0.008 | 0.034 | 0.008 | 0.034 | 0.007 | 0.023 |
| June | | | 0.044 | -0.395 | 0.096 | -0.389 | 0.138 | -0.427 |
| | | | 0.008 | 0.034 | 0.008 | 0.034 | 0.008 | 0.023 |
| July | | | 0.067 | -0.321 | 0.144 | -0.312 | 0.130 | -0.354 |
| | | | 0.008 | 0.033 | 0.008 | 0.033 | 0.008 | 0.023 |
| August | | | 0.047 | -0.178 | 0.084 | -0.166 | 0.110 | -0.245 |
| | | | 0.008 | 0.032 | 0.008 | 0.032 | 0.008 | 0.022 |
| September | | | 0.034 | -0.202 | 0.031 | -0.202 | 0.043 | -0.208 |
| | | | 0.008 | 0.033 | 0.008 | 0.033 | 0.008 | 0.022 |
| October | | | 0.268 | -0.117 | 0.279 | -0.116 | 0.188 | -0.158 |
| | | | 0.008 | 0.032 | 0.008 | 0.032 | 0.007 | 0.022 |
| November | | | 0.328 | -0.107 | 0.301 | -0.109 | 0.216 | -0.142 |
| | | | 0.008 | 0.032 | 0.008 | 0.032 | 0.007 | 0.022 |
| December | | | 0.222 | 0.054 | 0.223 | 0.050 | 0.173 | 0.014 |
| | | | 0.008 | 0.031 | 0.008 | 0.031 | 0.007 | 0.021 |
| **Cohort Year indicators** | | | | | | | | |
| Cohort Year 1995 | | | -7.50 | -11.63 | -11.77 | -5.41 | -13.66 | -7.25 |
| | | | 0.03 | 0.19 | 0.07 | 0.22 | 0.17 | 0.20 |
| Cohort Year 1996 | | | -7.18 | -11.36 | -11.52 | -4.77 | -13.42 | -6.60 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.20 |
| Cohort Year 1997 | | | -6.76 | -11.53 | -11.24 | -4.73 | -13.18 | -6.54 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 1998 | | | -6.82 | -11.38 | -11.46 | -4.53 | -13.10 | -6.27 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 1999 | | | -6.66 | -10.72 | -11.39 | -4.16 | -13.14 | -5.99 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 2000 | | | -5.94 | -9.51 | -10.73 | -3.48 | -12.83 | -5.52 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 2001 | | | -5.59 | -10.08 | -10.82 | -3.72 | -12.71 | -5.60 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 2002 | | | -5.78 | -10.19 | -11.02 | -3.71 | -12.90 | -5.68 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 2003 | | | -6.58 | -10.69 | -11.60 | -4.04 | -12.97 | -6.01 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 2004 | | | -6.68 | -10.45 | -11.29 | -3.90 | -12.85 | -6.02 |
| | | | 0.02 | 0.18 | 0.07 | 0.21 | 0.17 | 0.19 |
| Cohort Year 2005 | | | -7.06 | -10.55 | -11.32 | -3.97 | -12.81 | -6.02 |
| | | | 0.03 | 0.18 | 0.07 | 0.21 | 0.17 | 0.20 |
| Cohort Year 2006 | | | -7.02 | -10.43 | -11.28 | -3.90 | -12.97 | -6.04 |
| | | | 0.0560 | 0.2540 | 0.0873 | 0.2780 | 0.1768 | 0.2340 |

**Table Continued**

Notes: All estimateion runs utilized 15459583 observations.

**Table 3: Continued: Multinomial Logit Model Estimates of Defualt & Prepayment**

(Standard Errors Under Estimates)

| | Age Spline only | | Age Spline, Cohort & Transaction Month | | Full Model Less UPB & Refi/Burnout | | Full Model | |
|---|---|---|---|---|---|---|---|---|
| Log Likelihood | -2641381.9 | | -1.95E+06 | | -1894401.8 | | 605858 | |
| LR ChiSquared | 8.69E+06 | | 3.01E+07 | | 3.02E+07 | | 1.01E+07 | |
| ChiSquared d.o.f | 24 | | 70 | | 83 | | 101 | |
| Pseud R2 | 0.8445 | | 0.8853 | | 0.8885 | | 0.8931 | |
| **Original LTV Spline** | **Prepay** | **Default** | **Prepay** | **Default** | **Prepay** | **Default** | **Prepay** | **Default** |
| 0 < OLTV <70 | | | | | 0.0036 | 0.0011 | -0.0008 | 0.0081 |
| | | | | | 0.0002 | 0.0009 | 0.0002 | 0.0008 |
| 70 < OLTV < 81 | | | | | -0.0019 | 0.0193 | -0.0024 | 0.0160 |
| | | | | | 0.0005 | 0.0021 | 0.0004 | 0.0015 |
| 81 < OLTV | | | | | -0.0070 | 0.0321 | -0.0053 | 0.0230 |
| | | | | | 0.0003 | 0.0012 | 0.0003 | 0.0008 |
| **Credit Score Spline** | | | | | | | | |
| 450 < Fico < 635 | | | | | 0.614 | -0.939 | 0.198 | -0.727 |
| | | | | | 0.011 | 0.017 | 0.007 | 0.012 |
| 635 < Fico < 695 | | | | | 0.426 | -2.217 | 0.209 | -1.755 |
| | | | | | 0.011 | 0.043 | 0.009 | 0.034 |
| 695 < Fico < 850 | | | | | 0.060 | -1.941 | -0.012 | -1.742 |
| | | | | | 0.006 | 0.057 | 0.006 | 0.048 |
| **CMT2 to CMT10 Slope** | | | | | | | | |
| Yield Curve Slope | | | | | 0.538 | | 0.216 | |
| | | | | | 0.002 | | 0.002 | |
| **Oringal UPB Spline** | | | | | | | | |
| Original UPB Spline (50k < OUPB) | | | | | | | 0.0117 | -0.0100 |
| | | | | | | | 0.0006 | 0.0009 |
| Original UPB Spline (50k,150k) | | | | | | | 0.0067 | -0.0026 |
| | | | | | | | 0.0001 | 0.0002 |
| Original UPB Spline (150k,250k) | | | | | | | 0.0023 | -0.0052 |
| | | | | | | | 0.0001 | 0.0002 |
| Original UPB Spline (250k>OUPB) | | | | | | | -0.0001 | -0.0008 |
| | | | | | | | 0.0000 | 0.0001 |

**Table 3: Continued: Multinomial Logit Model Estimates of Defualt & Prepayment: Refinance-Burnout Model for the Full Model**

(Standard Errors Under Estimates)

| | yt_i_0 | yt_i_1 | yt_i_2 | yt_i_3 | yt_i_4 | yt_i_5 |
|---|---|---|---|---|---|---|
| **yt_1_j** | 3.535 | 5.195 | 5.553 | 4.765 | 0.789 | 0.000 |
| | 0.181 | 0.058 | 0.152 | 0.208 | 0.539 | 0.000 |
| **yt_2_j** | 2.676 | 12.114 | 0.674 | 3.493 | -0.176 | 4.767 |
| | 0.198 | 0.641 | 0.755 | 0.416 | 0.297 | 0.260 |
| **yt_3_j** | 2.715 | 9.319 | 8.952 | 1.431 | 0.550 | 2.965 |
| | 0.223 | 0.974 | 1.022 | 0.603 | 0.433 | 0.168 |
| **yt_4_j** | 2.835 | 5.949 | 14.811 | 5.636 | -2.954 | 1.873 |
| | 0.295 | 1.677 | 1.673 | 0.997 | 0.516 | 0.175 |
| **yt_5_j** | 4.514 | -5.485 | 15.487 | 4.313 | 2.383 | -0.148 |
| | 0.357 | 2.070 | 1.304 | 0.824 | 0.479 | 0.061 |

**Sample Descriptive Statistics**

| | Obs | Mean | Std. Dev | Min | Max |
|---|---|---|---|---|---|
| **Seasoning Spline Variables** | | | | | |
| age1 [0 < Months < 3) | 15459583 | 2.794 | 0.662 | 0 | 3 |
| age2 (3=< Months < 6) | 15459583 | 2.493 | 1.064 | 0 | 3 |
| age3 (6=< Months <9) | 15459583 | 2.210 | 1.275 | 0 | 3 |
| age4 (9=< Months < 12) | 15459583 | 1.948 | 1.392 | 0 | 3 |
| age5 (12 =< Months < 18) | 15459583 | 3.213 | 2.852 | 0 | 6 |
| age6 (18 =< Months < 24) | 15459583 | 2.458 | 2.837 | 0 | 6 |
| age7 (24 =< Months < 30) | 15459583 | 1.842 | 2.667 | 0 | 6 |
| age8 (30 =< Months < 40) | 15459583 | 1.987 | 3.737 | 0 | 10 |
| age9 (40 =< Months < 50) | 15459583 | 1.036 | 2.843 | 0 | 10 |
| age10 (50 =< Months < 60) | 15459583 | 0.491 | 2.013 | 0 | 10 |
| age11 (60 =< Months < 90) | 15459583 | 0.390 | 2.745 | 0 | 30 |
| age12 (90 =< Months ) | 15459583 | 0.037 | 0.863 | 0 | 49 |
| | | | | | |
| **Transaction Month** | | | | | |
| February | 15459583 | 0.083 | 0.276 | 0 | 1 |
| March | 15459583 | 0.083 | 0.276 | 0 | 1 |
| April | 15459583 | 0.084 | 0.277 | 0 | 1 |
| May | 15459583 | 0.084 | 0.278 | 0 | 1 |
| June | 15459583 | 0.085 | 0.279 | 0 | 1 |
| July | 15459583 | 0.086 | 0.280 | 0 | 1 |
| August | 15459583 | 0.086 | 0.281 | 0 | 1 |
| September | 15459583 | 0.081 | 0.272 | 0 | 1 |
| October | 15459583 | 0.082 | 0.274 | 0 | 1 |
| November | 15459583 | 0.082 | 0.274 | 0 | 1 |
| December | 15459583 | 0.082 | 0.275 | 0 | 1 |
| | | | | | |
| **Cohort Year (year of first PMT)** | | | | | |
| Cohort Year 1995 | 15459583 | 0.018 | 0.132 | 0 | 1 |
| Cohort Year 1996 | 15459583 | 0.050 | 0.217 | 0 | 1 |
| Cohort Year 1997 | 15459583 | 0.106 | 0.308 | 0 | 1 |
| Cohort Year 1998 | 15459583 | 0.279 | 0.449 | 0 | 1 |
| Cohort Year 1999 | 15459583 | 0.167 | 0.373 | 0 | 1 |
| Cohort Year 2000 | 15459583 | 0.068 | 0.252 | 0 | 1 |
| Cohort Year 2001 | 15459583 | 0.082 | 0.274 | 0 | 1 |
| Cohort Year 2002 | 15459583 | 0.067 | 0.250 | 0 | 1 |
| Cohort Year 2003 | 15459583 | 0.094 | 0.292 | 0 | 1 |
| Cohort Year 2004 | 15459583 | 0.039 | 0.193 | 0 | 1 |
| Cohort Year 2005 | 15459583 | 0.026 | 0.159 | 0 | 1 |
| Cohort Year 2006 | 15459583 | 0.006 | 0.076 | 0 | 1 |

**Continued . . .**

**Continued: Sample Descriptive Statistics**

| Variable | Obs | Mean | Std. Dev | Min | Max |
|---|---|---|---|---|---|
| Yield Curve Slope | 15459583 | 0.854 | 1.012 | -4.02 | 2.59 |
| | | | | | |
| **Credit Score Spline** | | | | | |
| Fico Score/100 | 15459583 | 7.028 | 0.650 | 4 | 8.48 |
| 450 < Fico < 635 | 15459583 | 6.277 | 0.236 | 4 | 6.35 |
| 635 < Fico < 695 | 15459583 | 0.448 | 0.231 | 0 | 0.6 |
| 695 < Fico < 850 | 15459583 | 0.303 | 0.333 | 0 | 1.529999 |
| | | | | | |
| **Original LTV Spline** | | | | | |
| Original UPB Spline (50k < OUPB) | 15459583 | 49.469 | 3.030 | 8.8 | 50 |
| Original UPB Spline (50k,150k) | 15459583 | 80.928 | 32.807 | 0 | 100 |
| Original UPB Spline (150k,250k) | 15459583 | 61.090 | 46.358 | 0 | 100 |
| Original UPB Spline (250k>OUPB) | 15459583 | 69.977 | 116.085 | 0 | 3520 |
| | | | | | |
| **Refinance Burnout Spline** | | | | | |
| yt_1_0 | 15459583 | 0.753 | 0.325 | 0 | 0.9 |
| yt_2_0 | 15459583 | 0.055 | 0.215 | 0 | 0.9 |
| yt_3_0 | 15459583 | 0.028 | 0.155 | 0 | 0.9 |
| yt_4_0 | 15459583 | 0.019 | 0.128 | 0 | 0.9 |
| yt_5_0 | 15459583 | 0.040 | 0.186 | 0 | 0.9 |
| yt_1_1 | 15459583 | 0.072 | 0.060 | 0 | 0.15 |
| yt_2_1 | 15459583 | 0.009 | 0.035 | 0 | 0.15 |
| yt_3_1 | 15459583 | 0.004 | 0.025 | 0 | 0.15 |
| yt_4_1 | 15459583 | 0.003 | 0.021 | 0 | 0.15 |
| yt_5_1 | 15459583 | 0.007 | 0.031 | 0 | 0.15 |
| yt_1_2 | 15459583 | 0.006 | 0.015 | 0 | 0.05 |
| yt_2_2 | 15459583 | 0.003 | 0.011 | 0 | 0.05 |
| yt_3_2 | 15459583 | 0.001 | 0.008 | 0 | 0.05 |
| yt_4_2 | 15459583 | 0.001 | 0.007 | 0 | 0.05 |
| yt_5_2 | 15459583 | 0.002 | 0.010 | 0 | 0.05 |
| yt_1_3 | 15459583 | 0.002 | 0.009 | 0 | 0.05 |
| yt_2_3 | 15459583 | 0.002 | 0.010 | 0 | 0.05 |
| yt_3_3 | 15459583 | 0.001 | 0.007 | 0 | 0.05 |
| yt_4_3 | 15459583 | 0.001 | 0.006 | 0 | 0.05 |
| yt_5_3 | 15459583 | 0.002 | 0.010 | 0 | 0.05 |
| yt_1_4 | 15459583 | 0.000 | 0.002 | 0 | 0.05 |
| yt_2_4 | 15459583 | 0.001 | 0.007 | 0 | 0.05 |
| yt_3_4 | 15459583 | 0.001 | 0.006 | 0 | 0.05 |
| yt_4_4 | 15459583 | 0.001 | 0.005 | 0 | 0.05 |
| yt_5_4 | 15459583 | 0.002 | 0.009 | 0 | 0.05 |
| yt_1_5 | 15459583 | 0.000 | 0.000 | 0 | 0.0153237 |
| yt_2_5 | 15459583 | 0.000 | 0.004 | 0 | 0.1021033 |
| yt_3_5 | 15459583 | 0.001 | 0.008 | 0 | 0.1537285 |
| yt_4_5 | 15459583 | 0.001 | 0.007 | 0 | 0.1938814 |
| yt_5_5 | 15459583 | 0.004 | 0.027 | 0 | 0.5590822 |